Math 18: Lectures ABC **Midterm Exam 1 v. 1** **Name:** ____________________

January 31, 2018 **(Total Points: *30*)** **PID:** ____________________

Instructions

1. **Write your Name and PID in the spaces provided above.**
2. **Make sure your Name is on every page.**
3. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
4. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
5. **Read each question carefully and answer each question completely.**
6. **Write your solutions clearly in the spaces provided; only work on this exam booklet will be graded.**
7. **Show all of your work. No credit will be given for unsupported answers, even if correct.**

(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(8 points) 1. Let A be any 10×8 matrix. For each statement about A, circle **T** if it is *always* True; circle **F** if it is *ever* False. 2 points will be assigned for each correct response, 1 point for each blank non-response, and 0 points for each incorrect response. No justification is required.

(T F) The matrix equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b} \in \mathbb{R}^{10}$.

(T F) The homogeneous system $A\mathbf{x} = \mathbf{0}$ is consistent.

(T F) The columns of A are linearly independent.

(T F) $A(2\mathbf{x}) = 2A\mathbf{x}$ for all vectors $\mathbf{x} \in \mathbb{R}^8$.

(9 points) 2.

The matrix $A = \begin{bmatrix} 1 & 2 & 0 & 1 & 2 \\ 2 & 4 & 1 & 4 & 3 \\ 1 & 2 & -1 & -1 & 3 \end{bmatrix}$ has reduced row-echelon form $\begin{bmatrix} 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Describe the general solution to the vector equation $x_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$.

(b) Do the columns of A span $\mathbb{R}^3$? Briefly explain your answer.

(c) Are the first and fourth columns of A linearly independent? Briefly explain your answer.

(6 points) 3. Consider the following system of equations.

$$\begin{aligned} x_1 - x_2 &= h \\ x_1 + hx_2 &= 4h \\ 2x_1 - 2x_2 &= 4 \end{aligned}$$

(a) Find the unique value of h for which the system is consistent, and determine the solution set in that case.

(b) Consider the corresponding *homogeneous* system with the same coefficient matrix. Find the unique value of h for which this system has infinitely many solutions, and describe the solution set in parametric form in this case.

(6 points) 4. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be three vectors in $\mathbb{R}^4$, with the following properties: $\{\mathbf{u}, \mathbf{v}\}$ are linearly independent, and $\mathbf{w}$ is *not* in the span generated by $\mathbf{u}$ and $\mathbf{v}$.

(a) Explain why $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ are, in fact, linearly independent. [Hint: It might be easier to explain why it is impossible for the vectors to be linearly *dependent*.]

(b) Let $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation satisfying

$$T(\mathbf{u}) + T(\mathbf{v}) = T(2\mathbf{u} + \mathbf{w}).$$

Is T one-to-one? Justify your answer.