Math 142A **Midterm Exam 1 v. A** **Name:** ____________________

October 25, 2017 **(Total Points: *25*)** **PID:** ____________________

Instructions

1. **Write your Name and PID in the spaces provided above.**
2. **Make sure your Name is on every page.**
3. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
4. **Put away ANY devices that can be used for communication or can access the Internet.**
5. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
6. **Read each question carefully and answer each question completely.**
7. **Write your solutions clearly in the spaces provided.**
8. **Show all of your work. No credit will be given for unsupported answers, even if correct.**

(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Let $\{a_n\}$ be a sequence.

(a) Prove that if $\{a_n\}$ converges, then $\{|a_n|\}$ converges.

(b) Exhibit an example for which $\{|a_n|\}$ converges, but $\{a_n\}$ diverges.

(6 points) 2. Given a sequence $\{a_n\}$ with $\lim_{n\to\infty} a_n = a$. Show that if $\{a_{n_k}\}$ is a subsequence of $\{a_n\}$, then $\lim_{k\to\infty} a_{n_k} = a$.

(6 points) 3. Let $\{a_n\}$ be a sequence. Prove that $\{a_n\}$ is bounded if and only if there is an interval $[c, d]$ such that $\{a_n\}$ is a sequence in $[c, d]$.

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(6 points) 4. Define $\{a_n\}$ by $\begin{cases} a_1 &= \sqrt{2}, \\ a_{n+1} &= \sqrt{2+a_n} \text{ for } n \geq 1. \end{cases}$

(a) Show that $a_n \leq 2$ for all indices n.

(b) Show that $a_{n+1} > a_n$ for all indices n.

(c) State how you know that $\{a_n\}$ converges.

(d) Determine the value of $\lim\limits_{n\to\infty} a_n$.