0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. (6 points) Let \( \{a_n\} \) be a sequence.
   
   (a) Prove that if \( \{a_n\} \) converges, then \( \{|a_n|\} \) converges.

(b) Exhibit an example for which \( \{|a_n|\} \) converges, but \( \{a_n\} \) diverges.
(6 points) 2. Given a sequence \( \{a_n\} \) with \( \lim_{n \to \infty} a_n = a \). Show that if \( \{a_{n_k}\} \) is a subsequence of \( \{a_n\} \), then \( \lim_{k \to \infty} a_{n_k} = a \).
3. Let \( \{a_n\} \) be a sequence. Prove that \( \{a_n\} \) is bounded if and only if there is an interval \([c,d]\) such that \( \{a_n\} \) is a sequence in \([c,d]\).
4. Define \( \{a_n\} \) by \[
\begin{align*}
a_1 &= \sqrt{2}, \\
a_{n+1} &= \sqrt{2 + a_n} \quad \text{for } n \geq 1.
\end{align*}
\]

(a) Show that \( a_n \leq 2 \) for all indices \( n \).

(b) Show that \( a_{n+1} > a_n \) for all indices \( n \).

(c) State how you know that \( \{a_n\} \) converges.

(d) Determine the value of \( \lim_{n \to \infty} a_n \).