

A List of Tautologies

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|-----|--|------------------------|
| 1. | $P \vee \neg P$ | |
| 2. | $\neg(P \wedge \neg P)$ | |
| 3. | $P \rightarrow P$ | |
| 4. | a) $P \leftrightarrow (P \vee P)$
b) $P \leftrightarrow (P \wedge P)$ | idempotent laws |
| 5. | $\neg\neg P \leftrightarrow P$ | double negation |
| 6. | a) $(P \vee Q) \leftrightarrow (Q \vee P)$
b) $(P \wedge Q) \leftrightarrow (Q \wedge P)$
c) $(P \leftrightarrow Q) \leftrightarrow (Q \leftrightarrow P)$ | commutative laws |
| 7. | a) $(P \vee (Q \vee R)) \leftrightarrow ((P \vee Q) \vee R)$
b) $(P \wedge (Q \wedge R)) \leftrightarrow ((P \wedge Q) \wedge R)$ | associative laws |
| 8. | a) $(P \wedge (Q \vee R)) \leftrightarrow ((P \wedge Q) \vee (P \wedge R))$
b) $(P \vee (Q \wedge R)) \leftrightarrow ((P \vee Q) \wedge (P \vee R))$ | distributive laws |
| 9. | a) $(P \vee \mathcal{C}) \leftrightarrow P$
b) $(P \wedge \mathcal{C}) \leftrightarrow \mathcal{C}$
c) $(P \vee \mathcal{T}) \leftrightarrow \mathcal{T}$
d) $(P \wedge \mathcal{T}) \leftrightarrow P$ | identity laws |
| 10. | a) $\neg(P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)$
b) $\neg(P \vee Q) \leftrightarrow (\neg P \wedge \neg Q)$ | DeMorgan's laws |
| 11. | a) $(P \leftrightarrow Q) \leftrightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P))$
b) $(P \leftrightarrow Q) \leftrightarrow ((P \wedge Q) \vee (\neg P \wedge \neg Q))$
c) $(P \leftrightarrow Q) \leftrightarrow (\neg P \leftrightarrow \neg Q)$ | equivalence |
| 12. | a) $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$
b) $\neg(P \rightarrow Q) \leftrightarrow (P \wedge \neg Q)$ | implication |
| 13. | $(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$ | contrapositive |
| 14. | $(P \rightarrow Q) \leftrightarrow ((P \wedge \neg Q) \rightarrow \mathcal{C})$ | reductio ad absurdum |
| 15. | a) $((P \rightarrow R) \wedge (Q \rightarrow R)) \leftrightarrow ((P \vee Q) \rightarrow R)$
b) $((P \rightarrow Q) \wedge (P \rightarrow R)) \leftrightarrow (P \rightarrow (Q \wedge R))$
c) $((P \rightarrow Q) \vee (P \rightarrow R)) \leftrightarrow (P \rightarrow (Q \vee R))$ | |
| 16. | $((P \wedge Q) \rightarrow R) \leftrightarrow (P \rightarrow (Q \rightarrow R))$ | exportation law |
| 17. | $P \rightarrow (P \vee Q)$ | addition |
| 18. | $(P \wedge Q) \rightarrow P$ | simplification |
| 19. | $(P \wedge (P \rightarrow Q)) \rightarrow Q$ | modus ponens |
| 20. | $((P \rightarrow Q) \wedge \neg Q) \rightarrow \neg P$ | modus tollens |
| 21. | $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$ | hypothetical syllogism |
| 22. | $((P \vee Q) \wedge \neg P) \rightarrow Q$ | disjunctive syllogism |
| 23. | $(P \rightarrow \mathcal{C}) \rightarrow \neg P$ | absurdity |
| 24. | $((P \rightarrow Q) \wedge (R \rightarrow S)) \rightarrow ((P \vee R) \rightarrow (Q \vee S))$ | |
| 25. | $(P \rightarrow Q) \rightarrow ((P \vee R) \rightarrow (Q \vee R))$ | |

Notes

1. \mathcal{T} refers to any statement which is a tautology.
2. \mathcal{C} refers to any statement which is a contradiction.
3. Item 21 is often called "transitivity".