

Instructions

1. Write your Name and PID in the spaces provided above.
 2. Complete the *Excel with Integrity Pledge* on the last page.
 3. Make sure your Name is on every page you turn in.
 4. No calculators, tablets, phones, or other electronic devices are allowed during this exam.
 5. Put away ANY devices that can be used for communication or can access the Internet.
 6. You may use one handwritten page of notes, but no books or other assistance during this exam.
 7. Read each question carefully and answer each question completely.
 8. Write your solutions clearly in the spaces provided.
 9. Show all of your work. No credit will be given for unsupported answers, even if correct.
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- (2 points) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. For each of the following, find an example of a function that satisfies the stated properties and briefly explain how each example satisfies the required properties.

(3 points)

- (a) A non-constant function $f : \mathbb{R} \rightarrow \mathbb{R}$ that is uniformly continuous. Make sure your example function is defined for all $x \in \mathbb{R}$. [Hint: Your example need not be complicated.]

(3 points)

- (b) A continuous function $g : (0, 1) \rightarrow \mathbb{R}$ that is not uniformly continuous.

- (6 points) 2. Let S be a set that is bounded below but has no minimum, and let λ be the infimum of S . Prove that there is a sequence (s_n) in S converging to λ .

(6 points) 3. Let $(a_n)_{n \in \mathbb{N}}$ be a sequence such that the sequence $(n^2 a_n)_{n \in \mathbb{N}}$ converges.

Show that the series $\sum_{n=1}^{\infty} a_n$ converges absolutely.

(6 points) 4. Let (a_n) be a convergent sequence with $a_n \geq 0$ for every index n , and let $a = \lim_{n \rightarrow \infty} a_n$.

Is $a \geq 0$? If so, prove it; if not, exhibit a counterexample.

5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \frac{\sqrt{1+3x^2}-1}{x^2} & \text{if } x \neq 0, \\ y_0 & \text{if } x = 0. \end{cases}$$

(2 points) (a) Write the definition of continuity at $x = 0$ for a function $f : \mathbb{R} \rightarrow \mathbb{R}$.

(6 points) (b) Determine the value of y_0 for which $f(x)$ is continuous at $x = 0$. Be sure to justify your computation.

(8 points) 6. (a) Write the definition of a Cauchy sequence.

(b) Prove that if $(a_n)_{n \in \mathbb{N}}$ is a sequence such that

$$|a_m - a_n| < \frac{1}{n} + \frac{1}{m}, \quad \text{for every } n, m \in \mathbb{N},$$

then $(a_n)_{n \in \mathbb{N}}$ is a Cauchy sequence.

(8 points) 7. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is *Lipschitz* if there exists a constant $C \geq 0$ such that

$$|f(x) - f(y)| \leq C |x - y| \text{ for all } x, y \in \mathbb{R}.$$

(a) Write the definition of uniform continuity for a function $f : \mathbb{R} \rightarrow \mathbb{R}$.

(b) Prove that a Lipschitz function $f : \mathbb{R} \rightarrow \mathbb{R}$ is uniformly continuous.

Math 142A Excel with Integrity Pledge

The Excel with Integrity pledge affirms the UC San Diego commitment to excel with integrity both on and off campus, in academic, professional, and research endeavors.

According to the International Center for Academic Integrity, academic integrity means having the courage to act in ways that are honest, fair, responsible, respectful & trustworthy even when it is difficult. Creating work with integrity is important because otherwise we are misrepresenting our knowledge and abilities and the University is falsely certifying our accomplishments. And when this happens, the UCSD degree loses its value and we've all wasted our time and talents!

Name: _____ PID: _____

Excel with Integrity Pledge

*I am **fair** to my classmates and instructors by not using any unauthorized aids.*

*I **respect** myself and my university by upholding educational and evaluative goals.*

*I am **honest** in my representation of myself and of my work.*

*I accept **responsibility** for ensuring my actions are in accord with academic integrity.*

*I show that I am **trustworthy** even when no one is watching.*

Affirm your adherence to this pledge by writing the following statement in the space below:

I Excel with Integrity.

Signature: _____ Date: _____