
Instructions

1. Write your Name and PID in the spaces provided above.
 2. Complete the *Excel with Integrity Pledge* on the last page.
 3. Make sure your Name is on every page.
 4. No calculators, tablets, phones, or other electronic devices are allowed during this exam.
 5. Put away ANY devices that can be used for communication or can access the Internet.
 6. You may use one handwritten page of notes, but no books or other assistance during this exam.
 7. Read each question carefully and answer each question completely.
 8. Write your solutions clearly in the spaces provided.
 9. Show all of your work. No credit will be given for unsupported answers, even if correct.
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(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. (a) Prove that an absolutely convergent series converges.

(b) Give an example of a series that converges, but does not absolutely converge. Be sure to justify why your example converges and why it does not absolutely converge.

(6 points) 2. Let (s_n) be the sequence defined by $s_n = (-1)^n \left(1 + \frac{1}{n}\right)$.

(a) Find $\limsup s_n$. *Briefly* explain how your answer fulfills the definition of $\limsup s_n$.

(b) Find $\liminf s_n$. *Briefly* explain how your answer fulfills the definition of $\liminf s_n$.

(c) Does (s_n) converge? Explain.

(6 points) 3. For each of the following *false* statements, exhibit a counterexample. Be sure to *briefly* state why each counterexample shows the corresponding statement to be false.

(a) If the function $f + g : \mathbb{R} \rightarrow \mathbb{R}$ is continuous, then so are the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$.

(b) Every (not necessarily continuous) function $f : [0, 1] \rightarrow \mathbb{R}$ has a maximum.

(c) Every continuous function $f : (0, 1) \rightarrow \mathbb{R}$ has a bounded image.

(6 points) 4. A function $f : D \rightarrow \mathbb{R}$ is said to be *Lipschitz* provided that there is a number $C \geq 0$ with

$$|f(u) - f(v)| \leq C |u - v| \quad \text{for all } u \text{ and } v \text{ in } D.$$

Show that a Lipschitz function is continuous.

Math 142A Excel with Integrity Pledge

The Excel with Integrity pledge affirms the UC San Diego commitment to excel with integrity both on and off campus, in academic, professional, and research endeavors.

According to the International Center for Academic Integrity, academic integrity means having the courage to act in ways that are honest, fair, responsible, respectful & trustworthy even when it is difficult. Creating work with integrity is important because otherwise we are misrepresenting our knowledge and abilities and the University is falsely certifying our accomplishments. And when this happens, the UCSD degree loses its value and we've all wasted our time and talents!

Name: _____ PID: _____

Excel with Integrity Pledge

*I am **fair** to my classmates and instructors by not using any unauthorized aids.*

*I **respect** myself and my university by upholding educational and evaluative goals.*

*I am **honest** in my representation of myself and of my work.*

*I accept **responsibility** for ensuring my actions are in accord with academic integrity.*

*I show that I am **trustworthy** even when no one is watching.*

Affirm your adherence to this pledge by writing the following statement in the space below:

I Excel with Integrity.

Signature: _____ Date: _____

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