

Math 142A Homework Assignment 3
Due 11:00pm Friday, October 23, 2026

1. Let (r_n) be an enumeration of the set $\mathbb{Q}$ of all rational numbers. Show there exists a subsequence (r_{n_k}) such that $\lim_{k \rightarrow \infty} r_{n_k} = +\infty$.
2. Prove that $\liminf s_n = -\limsup(-s_n)$ for every sequence (s_n) .
[Hint: See Definition 10.6 and Exercise 5.4]
3. Let S be a bounded set. Prove that there is an increasing sequence (s_n) in S such that $\lim s_n = \sup S$. Explain why if $\sup S$ is in S , then it suffices to define $s_n = \sup S$ for all n .
4. Let (s_n) be an increasing sequence. Show that (s_n) has no dominant terms.
5. Let (s_n) be a strictly decreasing sequence; that is, $s_n > s_{n+1}$ for every $n \in \mathbb{N}$. Show that every term of (s_n) is a dominant term.
6. Show that a monotonically increasing sequence is bounded if it has a bounded subsequence.
7. Suppose the sequence (s_n) is monotonically increasing and that it has a convergent subsequence. Show that (s_n) converges.
8. Let $c > 0$. Consider the quadratic equation $x^2 - x - c = 0$, where $x > 0$. Define the sequence (x_n) recursively, as follows:

$$x_n = \begin{cases} 1 & \text{if } n = 1 \\ \sqrt{c + x_{n-1}} & \text{if } n > 1 \end{cases}$$

Prove that (x_n) converges monotonically to the solution of the above quadratic equation.