

Math 142A Homework Assignment 1
Due 11:00pm Friday, October 9, 2026

1. Show that $\sqrt{4 + 2\sqrt{3}} - \sqrt{3}$ is a rational number.
2. Find all rational solutions of the equation $x^8 - 4x^5 + 13x^3 - 7x + 1 = 0$. Be sure to explain how you know you found all the rational solutions.
3. (a) Show $|b| \leq a$ if and only if $-a \leq b \leq a$.
(b) Prove $||a| - |b|| \leq |a - b|$ for all $a, b \in \mathbb{R}$.
4. Let $a, b \in \mathbb{R}$. Show that if $a < b_1$ for every $b_1 > b$, then $a \leq b$.
5. Prove that if $a > 0$, then there exists $n \in \mathbb{N}$ such that $\frac{1}{n} < a < n$.
6. Let $a, b \in \mathbb{R}$. Show that if $a \leq b + \frac{1}{n}$ for all $n \in \mathbb{N}$, then $a \leq b$.
7. Let (t_n) be a bounded sequence; that is, there exists $M \geq 0$ such that $|t_n| \leq M$ for all n . Let (s_n) be a sequence such that $\lim s_n = 0$. Prove that $\lim(s_n t_n) = 0$.
8. Consider three sequences (a_n) , (b_n) , and (s_n) such that $a_n \leq s_n \leq b_n$ for all n , and $\lim a_n = \lim b_n = s$. Prove that $\lim s_n = s$.
9. Suppose (s_n) and (t_n) are sequences such that $|s_n| \leq t_n$ for all n and $\lim t_n = 0$. Prove that $\lim s_n = 0$.
10. Let (s_n) be a sequence that converges.
 - (a) Show that if $s_n \geq a$ for all but finitely many n , then $\lim s_n \geq a$.
 - (b) Show that if $s_n \leq b$ for all but finitely many n , then $\lim s_n \leq b$.
 - (c) Conclude that if all but finitely many s_n belong to $[a, b]$, then $\lim s_n$ belongs to $[a, b]$.