

Instructions

1. Write your Name and PID in the spaces provided above.
 2. Complete the *Excel with Integrity Pledge* on the last page.
 3. Make sure your Name is on every page and that your Integrity Pledge is completed correctly.
 4. No calculators, tablets, phones, or other electronic devices are allowed during this exam.
 5. Put away ANY devices that can be used for communication or can access the Internet.
 6. You may use one handwritten page of notes, but no books or other assistance during this exam.
 7. Read each question carefully and answer each question completely.
 8. Write your solutions clearly in the spaces provided. Work on scratch paper will not be graded.
 9. Show all of your work. No credit will be given for unsupported answers, even if correct.
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- (2 points) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Determine the set of all possible values of $\log \left[(1 + \sqrt{3}i)^i \right]$.

(6 points) 2. (a) Show that the series $\sum_{k=1}^{\infty} \frac{z^k}{k}$ has a radius of convergence $R = 1$.

(b) Explain why $\sum_{k=1}^{\infty} \frac{z^k}{k}$ does *not* converge uniformly for $|z| \leq 1$.

(6 points) 3. The function $f(z) = \frac{1}{z - \frac{z^2}{2}}$ is analytic on the annuli $A_1 = \{0 < |z| < 2\}$ and $A_2 = \{|z| > 2\}$.

(a) Determine the Laurent series expansion centered at $z = 0$ for $f(z) = \frac{1}{z - \frac{z^2}{2}}$ that converges on $A_1 = \{0 < |z| < 2\}$.

(b) Determine the Laurent series expansion centered at $z = 0$ for $f(z) = \frac{1}{z - \frac{z^2}{2}}$ that converges on $A_2 = \{|z| > 2\}$.

(6 points) 4. $g(z) = \sin\left(\frac{1}{z}\right)$ has an essential singularity at $z = 0$.

(a) Compute the residue of $g(z)$ at $z = 0$.

(b) Evaluate the integral $\oint_{|z|=2} g(z) dz$.

- (6 points) 5. Suppose $f(z)$ is a bounded entire function; that is, there is $M > 0$ such that (1) $|f(z)| \leq M$ for all $z \in \mathbb{C}$, and (2) $f(z)$ is analytic on the entire complex plane \mathbb{C} .
- (a) Given any $z \in \mathbb{C}$, Cauchy's integral formula asserts that

$$f'(z) = \frac{1}{2\pi i} \int_{|\zeta-z|=R} \frac{f(\zeta)}{(\zeta-z)^2} d\zeta \text{ for every } R > 0.$$

Using an ML -estimate and letting $R \rightarrow \infty$, show that $|f'(z)| = 0$.

- (b) Explain why this shows that any bounded entire function is constant.
[Recall: This amazing result is known as Liouville's theorem.]

(8 points) 6. Evaluate the improper integral

$$\int_{-\infty}^{\infty} \frac{1}{x^2 - 2x + 2} dx$$

as an appropriate limit of contour integrals, using the residue theorem. Be sure to clearly specify the contours you use and why the value of the integrals along the nonreal part of the contours tend to zero as $R \rightarrow \infty$.

- (10 points) 7. Answer each of the following multiple choice questions by circling the letter corresponding to your choice. Circle only one choice: Items with more than one choice circled will receive no credit, even if one of the choices is correct.

(These questions will be taken from the clicker questions you've seen during the term.)

Math 120A Excel with Integrity Pledge

The Excel with Integrity pledge affirms the UC San Diego commitment to excel with integrity both on and off campus, in academic, professional, and research endeavors.

According to the International Center for Academic Integrity, academic integrity means having the courage to act in ways that are honest, fair, responsible, respectful & trustworthy even when it is difficult. Creating work with integrity is important because otherwise we are misrepresenting our knowledge and abilities and the University is falsely certifying our accomplishments. And when this happens, the UCSD degree loses its value and we've all wasted our time and talents!

Name: _____ PID: _____

Excel with Integrity Pledge

*I am **fair** to my classmates and instructors by not using any unauthorized aids.*

*I **respect** myself and my university by upholding educational and evaluative goals.*

*I am **honest** in my representation of myself and of my work.*

*I accept **responsibility** for ensuring my actions are in accord with academic integrity.*

*I show that I am **trustworthy** even when no one is watching.*

Affirm your adherence to this pledge by writing the following statement in the space below:

I Excel with Integrity.

Signature: _____ Date: _____