Math 120A **Homework 6** **Due November 17, 2026**

1. Show that the zeros of $\sin(z)$ are all simple.
2. Show that $\cos(z+w) = \cos(z)\cos(w) - \sin(z)\sin(w)$, assuming the identity holds for z and w real.
3. Show that if the analytic function $f(z)$ has a zero of order N at z_0, then $f(z) = g(z)^N$ for some function $g(z)$ analytic near z_0 and satisfying $g'(z_0) \neq 0$.
4. For each of the following functions, find the Laurent expansion centered at $z = -1$ that converges at $z = \frac{1}{2}$, and determine the largest open set (annulus) on which the series converges.
   (a) $\dfrac{1}{z^2+z}$
   (b) $\dfrac{z-1}{z+1}$
5. Suppose that $f(z)$ is analytic on the punctured plane $D = \mathbb{C} \setminus \{0\}$. Show that there is a constant c such that $f(z) - \dfrac{c}{z}$ has a primitive in D. Provide a formula for c in terms of an integral of $f(z)$.

- Be sure to work out the additional exercises listed on the course homework page
  `https://mathweb.ucsd.edu/~jeggers/Math120A/homework.html`