1. Find the radius of convergence of each of the following series:
   (a) $\sum_{n=0}^{\infty} z^{3^n} = z + z^3 + z^9 + z^{27} + z^{81} + z^{243} \cdots$
   (b) $\sum_{p \text{ prime}} z^p = z^2 + z^3 + z^5 + z^7 + z^{11} + z^{13} + \cdots$

2. Define $f(z) := \mathrm{Log}(z)$.
   (a) Find the power series expansion of $f(z)$ about the point $z = i - 2$ and show that the radius of convergence is $R = \sqrt{5}$.
   (b) Explain why this does not contradict the fact that $\mathrm{Log}(z)$ has a discontinuity at $z = -2$.

3. Find the power series expansion of the principal branch $\mathrm{Tan}^{-1}(z)$ of the inverse tangent function about $z = 0$. What is the radius of convergence?

   Hint: Integrate the power series expansion of its derivative term by term.

4. Let E be a bounded subset of $\mathbb{C}$ over which area integrals can be defined, and set

   $$f(w) := \iint_E \frac{1}{w - z}\, dx\, dy; \quad \text{where } z = x + i\, y \text{ and } w \in \mathbb{C} \setminus E.$$

   Show that $f(w)$ is analytic at ∞, and find a formula for the coefficients of the power series of $f(w)$ at ∞ in descending powers of w.

   [Hint: Try a geometric series expansion.]

5. Define $f(z) = \dfrac{e^z}{1+z}$. Show that:
   (a) $f(z) = 1 + \frac{1}{2}z^2 - \frac{1}{3}z^3 + \frac{3}{8}z^4 - \frac{11}{30}z^5 + \cdots$.
   (b) The general n^{th} term of the power series is given by $a_n = (-1)^n \left[\frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!}\right]$, for $n \geq 2$.
   (c) What is the radius of convergence of the series?

6. Show that the zeros of $\sin(z)$ are all simple.

7. Show that $\cos(z + w) = \cos(z)\cos(w) - \sin(z)\sin(w)$, assuming the identity holds for z and w real.

8. Show that if the analytic function $f(z)$ has a zero of order N at z_0, then $f(z) = g(z)^N$ for some function $g(z)$ analytic near z_0 and satisfying $g'(z_0) \neq 0$.

9. For each of the following functions, find the Laurent expansion centered at $z = -1$ that converges at $z = \frac{1}{2}$, and determine the largest open set (disk) on which the series converges.
   (a) $\dfrac{1}{z^2 - z}$
   (b) $\dfrac{z-1}{z+1}$

10. Suppose that $f(z)$ is analytic on the punctured plane $D = \mathbb{C} \setminus \{0\}$. Show that there is a constant c such that $f(z) - \dfrac{c}{z}$ has a primitive in D. Provide a formula for c in terms of an integral of $f(z)$.