Math 102 Homework Assignment 1  
Due Thursday, January 13, 2022

1. The inverse of  
\[
\begin{bmatrix}
I & 0 & 0 \\
C & I & 0 \\
A & B & I
\end{bmatrix}
\]
is  
\[
\begin{bmatrix}
X & Y & I \\
Z & I & 0 \\
I & 0 & 0
\end{bmatrix}
\].

Find X, Y, and Z.

2. Let  
\[
A = \begin{bmatrix}
A_{11} & A_{12} \\
O & A_{22}
\end{bmatrix}
\]
with all four blocks are \(n \times n\) matrices and \(A_{11}\) and \(A_{22}\) nonsingular.

(a) Show that \(A\) is nonsingular and that \(A^{-1}\) is of the form  
\[
\begin{bmatrix}
A_{11}^{-1} & C \\
O & A_{22}^{-1}
\end{bmatrix}
\].

(b) Determine \(C\).

3. Let  
\[
\begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix}
,  \begin{bmatrix}
2 \\
2 \\
2
\end{bmatrix}
,  \begin{bmatrix}
2 \\
3 \\
4
\end{bmatrix}
\].

(a) Find the transition matrix corresponding to the change of basis from \(\mathcal{E} = [e_1, e_2, e_3]\) to \(\mathcal{U} = [u_1, u_2, u_3]\).

(b) Find the coordinates of each of the following vectors with respect to the basis \(\mathcal{U}\):

(i)  
\[
\begin{bmatrix}
2 \\
3 \\
4
\end{bmatrix}
\]

(ii)  
\[
\begin{bmatrix}
0 \\
0 \\
2
\end{bmatrix}
\]

4. (a) Find the transition matrix representing the change of coordinates on \(P_3\) for the ordered basis \(\mathcal{E} = [1, x, x^2]\) to the ordered basis \(\mathcal{D} = [1, 1 + x, 1 + x + x^2]\).

(b) Find the coordinates \([p]_\mathcal{D}\) for the polynomial \(p = 3 + 2x + x^2\) with respect to the ordered basis \(\mathcal{D}\).

5. Let \(L\) be a linear operator on \(\mathbb{R}^1\) such that \(L(1) = a\). Show that \(L(x) = ax\) for every \(x \in \mathbb{R}^1\).

6. Let \([v_1, \ldots, v_n]\) be a basis for a vector space \(V\), and let \(L_1\) and \(L_2\) be two linear transformations mapping \(V\) into a vector space \(W\). Show that if \(L_1(v_i) = L_2(v_i)\) for each \(i = 1, \ldots, n\), then \(L_1 = L_2\); that is, \(L_1(v) = L_2(v)\) for every \(v \in V\).