

Instructions

1. Write your Name and PID in the spaces provided above.
  2. Carefully read and complete the *Excel with Integrity Pledge* on the last page.
  3. Make sure your Name is on every page.
  4. No calculators, tablets, phones, or other electronic devices are allowed during this exam.
  5. Put away ANY devices that can be used for communication or can access the Internet.
  6. You may use one handwritten page of notes, but no books or other assistance during this exam.
  7. Read each question carefully and answer each question completely.
  8. Write your solutions clearly in the spaces provided.
  9. Show all of your work. No credit will be given for unsupported answers, even if correct.
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0. (2 points) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. (6 points) Evaluate the following integrals.

(a)  $\int_0^2 x e^{-2} dx$

(b)  $\int x^3 \sqrt{1 - x^2} dx$

2. (6 points)

(a) Does the following integral converge?  $\int_1^{\infty} \frac{1}{xe^x} dx$

(b) Use the integral test to determine whether the following series converges.  $\sum_{n=1}^{\infty} \frac{1}{ne^n}$

3. (8 points) Consider the following complex integral.

$$\int \frac{[\cos(\frac{x}{3}) + i \sin(\frac{x}{3})]^{12}}{[2 \cos(x) + 2i \sin(x)]^4} dx$$

- (a) Write the integral in complex exponential form.

- (b) Evaluate the integral. You may leave the result in complex exponential form.

4. (4 points) For each of the following series, determine whether it converges absolutely, converges conditionally, or diverges and then **circle your choice**. You do not need to justify your answer.

(a)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n + \sqrt{n}}$

Circle your choice:   **converges absolutely**      **converges conditionally**      **diverges**

(b)  $\sum_{n=1}^{\infty} \frac{n^2 \cdot 5^n}{n!}$

Circle your choice:   **converges absolutely**      **converges conditionally**      **diverges**

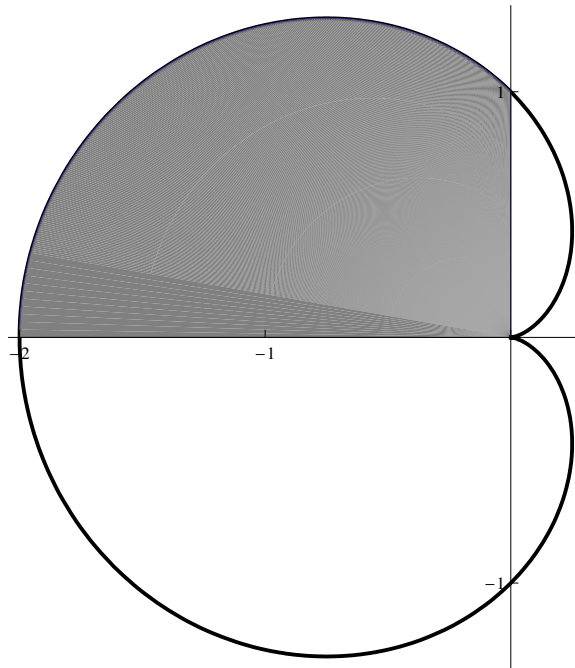
(c)  $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

Circle your choice:   **converges absolutely**      **converges conditionally**      **diverges**

(d)  $\sum_{n=1}^{\infty} \frac{2^n + n}{4^n + n^3 + 1}$

Circle your choice:   **converges absolutely**      **converges conditionally**      **diverges**

5. (4 points) Carefully write down an integral for the shaded portion of the cardioid  $r = 1 - \cos(\theta)$  shown in the figure below. You need not evaluate the integral.



6. (6 points) Find the interval of convergence of the following series.

$$\sum_{n=1}^{\infty} \frac{(x-6)^n}{\sqrt{n}3^n}$$

State which test(s) you used to support your steps.

7. (6 points) The following Taylor series expansion holds for all  $-1 < x < 1$ :

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}.$$

Use the Taylor series provided above to express the following integral as an infinite series.

$$\int_0^x \ln(1+t^2) dt$$

8. (8 points) Consider the polynomial  $p(x) = x^3 - 6x^2 + 13x - 6$ . Find constants  $a_0, a_1, a_2, a_3$  so that

$$p(x) = a_0 + a_1(x - 1) + a_2(x - 1)^2 + a_3(x - 1)^3.$$

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# Math 20B Excel with Integrity Pledge

The Excel with Integrity pledge affirms the UC San Diego commitment to excel with integrity both on and off campus, in academic, professional, and research endeavors.

According to the International Center for Academic Integrity, academic integrity means having the courage to act in ways that are honest, fair, responsible, respectful & trustworthy even when it is difficult. Creating work with integrity is important because otherwise we are misrepresenting our knowledge and abilities and the University is falsely certifying our accomplishments. And when this happens, the UCSD degree loses its value and we've all wasted our time and talents!

Name: \_\_\_\_\_ PID: \_\_\_\_\_

## Excel with Integrity Pledge

*I am **fair** to my classmates and instructors by not using any unauthorized aids.*

*I **respect** myself and my university by upholding educational and evaluative goals.*

*I am **honest** in my representation of myself and of my work.*

*I accept **responsibility** for ensuring my actions are in accord with academic integrity.*

*I show that I am **trustworthy** even when no one is watching.*

Affirm your adherence to this pledge by writing the following statement in the space below:

*I Excel with Integrity.*

Signature: \_\_\_\_\_ Date: \_\_\_\_\_