

Instructions

1. Write your Name and PID in the spaces provided above.
 2. Complete the *Excel with Integrity Pledge* on the last page.
 3. Make sure your Name is on every page you turn in.
 4. No calculators, tablets, phones, or other electronic devices are allowed during this exam.
 5. Put away ANY devices that can be used for communication or can access the Internet.
 6. You may use one handwritten page of notes, but no books or other assistance during this exam.
 7. Read each question carefully and answer each question completely.
 8. Write your solutions clearly in the spaces provided.
 9. Show all of your work. No credit will be given for unsupported answers, even if correct.
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- (2 points) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Let $f : (-1, 1) \rightarrow \mathbb{R}$ be a function such that f is differentiable at $x = 0$ and $f'(0) > 0$.

Prove that there is some $\delta > 0$ such that $f(x) > f(0)$ for all $x \in (0, \delta)$.

(6 points) 2. Let f be continuous on $\mathbb{R}$. Define $G(x) = \int_0^x (x-t)f(t) dt$ for all $x \in \mathbb{R}$.

Show that $G''(x) = f(x)$ for all $x \in \mathbb{R}$.

(8 points) 3. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = \sin(x)$.

(a) Determine a formula for the n^{th} Taylor polynomial $p_n(x)$ for $f(x)$ centered at $x_0 = \frac{\pi}{2}$.

(b) Show that $\lim_{n \rightarrow \infty} p_n(x) = f(x)$; that is, show that $\lim_{n \rightarrow \infty} [f(x) - p_n(x)] = 0$.

(8 points) 4. We know that every continuous function $f : [a, b] \rightarrow \mathbb{R}$ is integrable.

- (a) Let $g : [0, 1] \rightarrow \mathbb{R}$ be a continuous function. Let $P_n = \{0, \frac{1}{n}, \frac{2}{n}, \dots, \frac{n-1}{n}, 1\}$ be the n^{th} regular partition of $[0, 1]$. Show that

$$L(g, P_n) \leq \frac{1}{n} \sum_{k=1}^n g\left(\frac{k}{n}\right) \leq U(g, P_n) \text{ for every index } n.$$

- (b) Prove that $\lim_{n \rightarrow \infty} \left\{ \frac{1}{n} \sum_{k=1}^n g\left(\frac{k}{n}\right) \right\} = \int_0^1 g.$

- (6 points) 5. Using the properties of power series, determine the numerical value of the series $\sum_{k=1}^{\infty} \frac{1}{k \cdot 2^k}$.

(8 points) 6. Consider the sequence of functions (f_n) defined on $[0, 1]$ given by $f_n(x) = 2nx e^{-nx^2}$ for each $n \in \mathbb{N}$.

(a) Determine the limit function $f(x) = \lim_{n \rightarrow \infty} f_n(x)$.

(b) Show that $\int_0^1 f \neq \lim_{n \rightarrow \infty} \int_0^1 f_n$.

(c) Does f_n converge uniformly to f on $[0, 1]$? Briefly justify your answer.

- (6 points) 7. Let (f_n) be a sequence of monotonically increasing functions defined on a closed interval $[a, b]$ and converging pointwise to a function $f : [a, b] \rightarrow \mathbb{R}$.
Show that f is monotonically increasing on $[a, b]$.

Math 142B Excel with Integrity Pledge

The Excel with Integrity pledge affirms the UC San Diego commitment to excel with integrity both on and off campus, in academic, professional, and research endeavors.

According to the International Center for Academic Integrity, academic integrity means having the courage to act in ways that are honest, fair, responsible, respectful & trustworthy even when it is difficult. Creating work with integrity is important because otherwise we are misrepresenting our knowledge and abilities and the University is falsely certifying our accomplishments. And when this happens, the UCSD degree loses its value and we've all wasted our time and talents!

Name: _____ PID: _____

Excel with Integrity Pledge

*I am **fair** to my classmates and instructors by not using any unauthorized aids.*

*I **respect** myself and my university by upholding educational and evaluative goals.*

*I am **honest** in my representation of myself and of my work.*

*I accept **responsibility** for ensuring my actions are in accord with academic integrity.*

*I show that I am **trustworthy** even when no one is watching.*

Affirm your adherence to this pledge by writing the following statement in the space below:

I Excel with Integrity.

Signature: _____ Date: _____