

Instructions

1. Write your Name and PID in the spaces provided above.
 2. Complete the *Excel with Integrity Pledge* on the last page.
 3. Make sure your Name is on every page you turn in.
 4. No calculators, tablets, phones, or other electronic devices are allowed during this exam.
 5. Put away ANY devices that can be used for communication or can access the Internet.
 6. You may use one handwritten page of notes, but no books or other assistance during this exam.
 7. Read each question carefully and answer each question completely.
 8. Write your solutions clearly in the spaces provided.
 9. Show all of your work. No credit will be given for unsupported answers, even if correct.
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- (1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Let f be continuous on $\mathbb{R}$. Define $G(x) = \int_0^x (x-t)f(t) dt$ for all $x \in \mathbb{R}$.

Show that $G''(x) = f(x)$ for all $x \in \mathbb{R}$.

[Hint: Be sure to correctly apply the product rule when computing $G'(x)$.]

(6 points) 2. Let $f(x) = \begin{cases} 1 & \text{if } x = 0 \\ \frac{\sin(x)}{x} & \text{otherwise} \end{cases}$

(a) Show that $f(x)$ is continuous at $x = 0$.

(b) Show that the improper integral $\int_0^\infty f(x) dx$ converges.

- (6 points) 3. Not every uniformly convergent sequence of differentiable functions (f_n) defined on $\mathbb{R}$ converges to a differentiable function f on $\mathbb{R}$.

Consider the sequence of continuously differentiable functions (g_n) defined on $\mathbb{R}$ by

$$g_n(x) = \frac{1}{n} \arctan(n^2 x).$$

- (a) Show that (g_n) converges uniformly on $\mathbb{R}$ to a continuous function g defined on $\mathbb{R}$ and determine a formula for $g(x)$.

- (b) Show that the sequence $(g'_n(0))$ is unbounded. Briefly explain why this implies that (g'_n) does not converge pointwise to g' on $\mathbb{R}$. [Don't forget that $\arctan'(y) = \frac{1}{1+y^2}$.]

(6 points) 4. We have seen that the geometric series $\sum_{n=0}^{\infty} x^n$ converges to $\frac{1}{1-x}$ for every $x \in (-1, 1)$.

Fix a number ρ with $0 < \rho < 1$. Use the Weierstrass M-test to show that $\sum_{n=0}^{\infty} x^n$ converges *uniformly* for $x \in [-\rho, \rho]$.

Math 142B Excel with Integrity Pledge

The Excel with Integrity pledge affirms the UC San Diego commitment to excel with integrity both on and off campus, in academic, professional, and research endeavors.

According to the International Center for Academic Integrity, academic integrity means having the courage to act in ways that are honest, fair, responsible, respectful & trustworthy even when it is difficult. Creating work with integrity is important because otherwise we are misrepresenting our knowledge and abilities and the University is falsely certifying our accomplishments. And when this happens, the UCSD degree loses its value and we've all wasted our time and talents!

Name: _____ PID: _____

Excel with Integrity Pledge

*I am **fair** to my classmates and instructors by not using any unauthorized aids.*

*I **respect** myself and my university by upholding educational and evaluative goals.*

*I am **honest** in my representation of myself and of my work.*

*I accept **responsibility** for ensuring my actions are in accord with academic integrity.*

*I show that I am **trustworthy** even when no one is watching.*

Affirm your adherence to this pledge by writing the following statement in the space below:

I Excel with Integrity.

Signature: _____ Date: _____