

Math 142B Homework Assignment 3
Due 11:00pm Thursday, May 7, 2026

1. Define $f : \mathbb{R} \rightarrow \mathbb{R}$ by $f(t) = \begin{cases} 0 & \text{if } t < 0, \\ t & \text{if } 0 \leq t \leq 1, \\ 4 & \text{if } t > 1. \end{cases}$
 - (a) Determine the function $F(x) = \int_0^x f(t) dt$.
 - (b) Where is F continuous?
 - (c) Where is F differentiable? Compute $F'(x)$ at the points x where F is differentiable.
2. Let f be a continuous function on $\mathbb{R}$ and define $F(x) = \int_{x-1}^{x+1} f(t) dt$ for $x \in \mathbb{R}$.
Show that F is differentiable on $\mathbb{R}$ and compute F' .
3. Let f be a continuous function on $\mathbb{R}$ and define $G(x) = \int_0^{\sin(x)} f(t) dt$ for $x \in \mathbb{R}$.
Show that G is differentiable on $\mathbb{R}$ and compute G' .
4. Suppose f is a continuous function on $[a, b]$. Show that if $\int_a^b f(x)^2 dx = 0$, then $f(x) = 0$ for all $x \in [a, b]$.
5. Show that if f is a continuous real-valued function on $[a, b]$ satisfying $\int_a^b f(x)g(x) dx = 0$ for every continuous function g on $[a, b]$, then $f(x) = 0$ for all $x \in [a, b]$.
6. (a) Show that
 - i. $\int_0^1 x^{-p} dx = \frac{1}{1-p}$ if $0 < p < 1$.
 - ii. $\int_0^1 x^{-p} dx = +\infty$ if $p > 1$.(b) Show that $\int_0^\infty x^{-p} dx = +\infty$ for all $p > 0$.
7. Compute
 - (a) $\int_0^1 \log(x) dx$
 - (b) $\int_2^\infty \frac{\log(x)}{x} dx$
 - (c) $\int_0^\infty \frac{1}{1+x^2} dx$
8. Prove the following *comparison tests*. Let f and g be continuous functions on (a, b) such that $0 \leq f(x) \leq g(x)$ for all $x \in (a, b)$ and where a could be $-\infty$ and b could be $+\infty$.
 - (a) If $\int_a^b g(x) dx < \infty$, then $\int_a^b f(x) dx < \infty$.
 - (b) If $\int_a^b f(x) dx = +\infty$, then $\int_a^b g(x) dx = +\infty$.

9. (a) Using a comparison test, show that $\int_{-\infty}^{\infty} e^{-x^2} dx < \infty$.
- (b) Show that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$.
10. Suppose f is continuous on (a, b) , where a could be $-\infty$ and b could be $+\infty$. Show that if $\int_a^b |f(x)| dx < \infty$, then the integral $\int_a^b f(x) dx$ exists and is finite.