

1. Find the radius of convergence of each of the following series:

(a) $\sum_{n=0}^{\infty} z^{3^n} = z + z^3 + z^9 + z^{27} + z^{81} + z^{243} \dots$

(b) $\sum_{p \text{ prime}} z^p = z^2 + z^3 + z^5 + z^7 + z^{11} + z^{13} + \dots$

2. Define $f(z) := \text{Log}(z)$.

- (a) Find the power series expansion of $f(z)$ about the point $z = i - 2$ and show that the radius of convergence is $R = \sqrt{5}$.
- (b) Explain why this does not contradict the fact that $\text{Log}(z)$ has a discontinuity at $z = -2$.

3. Find the power series expansion of the principal branch $\text{Tan}^{-1}(z)$ of the inverse tangent function about $z = 0$. What is the radius of convergence?

Hint: Integrate the power series expansion of its derivative term by term.

4. Let E be a bounded subset of $\mathbb{C}$ over which area integrals can be defined, and set

$$f(w) := \iint_E \frac{1}{w - z} dx dy; \quad \text{where } z = x + iy \text{ and } w \in \mathbb{C} \setminus E.$$

Show that $f(w)$ is analytic at ∞ , and find a formula for the coefficients of the power series of $f(w)$ at ∞ in descending powers of w .

[Hint: Try a geometric series expansion.]

5. Define $f(z) = \frac{e^z}{1+z}$. Show that:

(a) $f(z) = 1 + \frac{1}{2}z^2 - \frac{1}{3}z^3 + \frac{3}{8}z^4 - \frac{11}{30}z^5 + \dots$

(b) The general n^{th} term of the power series is given by $a_n = (-1)^n \left[\frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^n}{n!} \right]$,
for $n \geq 2$.

- (c) What is the radius of convergence of the series?

- Be sure to work out the additional exercises listed on the course homework page

<https://mathweb.ucsd.edu/~jeggers/Math120A/homework.html>