

1. Show that an analytic function $f(z)$ has a primitive in D if and only if $\int_{\gamma} f(z) dz = 0$ for every closed path γ in D .
2. By integrating $e^{-\frac{z^2}{2}}$ around a rectangle $\mathcal{R}_t$ with vertices $-R$, R , $R + it$, $-R + it$, and sending $R \rightarrow \infty$; show that

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} e^{-itx} dx = e^{-\frac{t^2}{2}}, \quad -\infty < t < \infty.$$

Use the known value of the integral for $t = 0$.

3. Show that the series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

converges.

[Hint: Show that the partial sums of the series satisfy $S_2 < S_4 < S_6 < \cdots < S_5 < S_3 < S_1$.]

4. Show that

(a) $\sum_{k=2}^{\infty} \frac{1}{k \log(k)}$ diverges.

(b) $\sum_{k=2}^{\infty} \frac{1}{k (\log(k))^2}$ converges.

5. (a) Show that $\sum_{k=1}^{\infty} \frac{x^k}{k^2}$ converges uniformly for $|z| < 1$.

(b) Show that $\sum_{k=1}^{\infty} \frac{x^k}{k}$ does not converge uniformly for $|z| < 1$.

- Be sure to work out the additional exercises listed on the course homework page <https://mathweb.ucsd.edu/~jeggers/Math120A/homework.html>