

1. Suppose that P and Q are smooth functions on the annulus $\{z \in \mathbb{C} \mid a < |z| < b\}$ that satisfy $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$. Show directly using Green's theorem that $\oint_{|z|=r} P dx + Q dy$ is independent of the radius r , for $a < r < b$.

2. Show that if D is a bounded domain with smooth boundary, then

$$\int_{\partial D} \bar{z} dz = 2i \operatorname{Area}(D).$$

3. Let $P(x, y)$ and $Q(x, y)$ be continuous complex-valued functions on a curve γ , and define a function $F(w)$ on $\mathbb{C} \setminus \gamma$ by

$$F(w) := \int_{\gamma} \frac{P(x, y)}{z - w} dx + \int_{\gamma} \frac{Q(x, y)}{z - w} dy, \quad \text{where } z = x + iy.$$

- (a) Show that $F(w)$ is analytic for $w \in \mathbb{C} \setminus \gamma$.
(b) Express $F'(w)$ as a line integral over γ .
4. Suppose $h(z)$ is a continuous function on a curve γ . Show that

$$H(w) = \int_{\gamma} \frac{h(z)}{z - w} dz, \quad w \in \mathbb{C} \setminus \gamma,$$

is analytic on the complement of γ , $\mathbb{C} \setminus \gamma$. Find $H'(w)$.

5. Show that the differential

$$\frac{-y dx + x dy}{x^2 + y^2}, \quad (x, y) \neq (0, 0)$$

is closed. Show that it is not independent of path on any annulus centered at 0.

- Be sure to work out the additional exercises listed on the course homework page <https://mathweb.ucsd.edu/~jeggers/Math120A/homework.html>