Math 20E **Final Examination v. A** **Name:** ____________________

December 5, 2022 **(Total Points: *50*)** **PID:** ____________________

Instructions

1. **Write your Name and PID in the spaces provided above.**
2. **Complete the *Excel with Integrity Pledge* on the last page.**
3. **Make sure your Name is on every page.**
4. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
5. **Put away ANY devices that can be used for communication or can access the Internet.**
6. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
7. **Read each question carefully and answer each question completely.**
8. **Write your solutions clearly in the spaces provided. Scratch paper will not be accepted.**
9. **Show all of your work. No credit will be given for unsupported answers, even if correct.**

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0. (2 points) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. (8 points) Evaluate
$$\int_{y=0}^{1} \int_{x=\sqrt{y}}^{1} \sin\left(x^3\right)\, dx\, dy.$$
by changing the order of integration. (Note: you need not simplify trigonometric expressions in your final answer.)

2. (8 points) Use an appropriate change of variables to compute

$$\iint_D (x+y)\, e^{x-y}\, dx\, dy,$$

where D is the region bounded by the lines

$$\begin{aligned} x+y &= 0; \qquad & x-y &= 0; \\ x+y &= 2; \qquad & x-y &= 2. \end{aligned}$$

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3. (8 points) Consider the vector field

$$\mathbf{F}(x,y,z) = \left(e^{-yz}\cos(x),\ -ze^{-yz}\sin(x),\ -ye^{-yz}\sin(x)\right).$$

(a) Find a scalar function $f(x,y,z)$ such that $\mathbf{F} = \boldsymbol{\nabla} f$.

(b) Evaluate the line integral $\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s}$ from $(0,0,0)$ to $(\frac{\pi}{2},1,1)$ along the path $\mathbf{c}(t) = \left(\frac{\pi}{2}t,\ t^3\sin\left(\frac{\pi}{2}t\right),\ t^4\cos(2\pi t)\right)$.

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4. (8 points) Let S be the portion of the sphere $x^2 + y^2 + z^2 = 4$ with $z \geq \sqrt{2}$.

   (a) Find a parametrization for S. Be sure to clearly state the domain of your parametrization.

   (b) Find the area of S.

5. (8 points) Let $\mathbf{F}(x,y,z) = \left(3x + ze^{y^2},\ 5y + \sin\left(\frac{z}{x+2}\right),\ 2z - \tan^{-1}\left(xy^2+2\right)\right)$.

   Let S be the surface of the unit cube $[0,1]\times[0,1]\times[0,1]$ oriented with the outward-pointing unit normal vector.

   Use Gauss's theorem to compute the surface integral $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S}$.

6. (8 points) Let $\mathbf{F}(x, y, z) = (2y,\ -2x,\ zx^3y^2)$ and let S be the surface $x^2 + y^2 + 3z^2 = 1$ with $z \leq 0$ and oriented by the downward-pointing unit normal vector.

   Use Stokes' theorem to evaluate $\displaystyle\iint_S (\boldsymbol{\nabla}\times\mathbf{F})\cdot d\mathbf{S}$.

Math 20E
Formula Sheet

Vector Identities

1. $\nabla(f+g) = \nabla f + \nabla g$
2. $\nabla(cf) = c\,\nabla f$
3. $\nabla(fg) = f\,\nabla g + g\,\nabla f$
4. $\nabla\left(\frac{f}{g}\right) = \frac{g\,\nabla f - f\,\nabla g}{g^2}$, at points $\mathbf{x}$ where $g(\mathbf{x}) \neq 0$
5. $\nabla\cdot(\mathbf{F}+\mathbf{G}) = \nabla\cdot\mathbf{F} + \nabla\cdot\mathbf{G}$
6. $\nabla\times(\mathbf{F}+\mathbf{G}) = \nabla\times\mathbf{F} + \nabla\times\mathbf{G}$
7. $\nabla\cdot(f\mathbf{F}) = f\,\nabla\cdot\mathbf{F} + \mathbf{F}\cdot\nabla f$
8. $\nabla\times(f\mathbf{F}) = f\,\nabla\times\mathbf{F} + \nabla f\times\mathbf{F}$
9. $\nabla\cdot(\nabla\times\mathbf{F}) = 0$
10. $\nabla\times\nabla f = \mathbf{0}$
11. $\nabla\cdot(\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot(\nabla\times\mathbf{F}) - \mathbf{F}\cdot(\nabla\times\mathbf{G})$

Selected Integrals

1. $\int \tan(x)\,dx = \log|\sec(x)|$
2. $\int \sec(x)\,dx = \log|\sec(x)+\tan(x)|$
3. $\int \arcsin\left(\frac{x}{a}\right)\,dx = x\arcsin\left(\frac{x}{a}\right) + \sqrt{x^2+a^2}$
4. $\int \arctan\left(\frac{x}{a}\right)\,dx = x\arctan\left(\frac{x}{a}\right) - \frac{a}{2}\log(a^2+x^2)$
5. $\int \sin^2(x)\,dx = \frac{1}{2}\left(x - \sin(x)\cos(x)\right)$
6. $\int \cos^2(x)\,dx = \frac{1}{2}\left(x + \sin(x)\cos(x)\right)$
7. $\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \log(x+\sqrt{a^2+x^2})$
8. $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$
9. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\left(\frac{x}{a}\right)$
10. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right)$
11. $\int \frac{1}{\sqrt{x^2\pm a^2}}\,dx = \log(x+\sqrt{x^2\pm a^2})$
12. $\int \frac{1}{(a^2-x^2)^{3/2}}\,dx = \frac{x}{a^2\sqrt{a^2-x^2}}$
13. $\int \sqrt{x^2\pm a^2}\,dx = \frac{x}{2}\sqrt{x^2\pm a^2} \pm \frac{a^2}{2}\log(x+\sqrt{x^2\pm a^2})$
14. $\int (a^2-x^2)^{3/2}\,dx = \frac{x}{8}(5a^2-2x^2)\sqrt{a^2-x^2} + \frac{3a^4}{8}\arcsin\left(\frac{x}{a}\right)$

Path and Line Integrals over a Parameterized Curve (Path) $\mathbf{c}(t)$

1. Path integral of a scalar function f: $\displaystyle\int_{\mathbf{c}} f\,ds = \int_a^b f(\mathbf{c}(t))\,||\mathbf{c}'(t)||\,dt$
2. Line integral of a vector field $\mathbf{F}$: $\displaystyle\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{c}(t))\cdot\mathbf{c}'(t)\,dt$

Surface Integrals over a Parameterized Surface $\mathbf{\Phi}(u,v)$

1. Integral of a scalar function f: $\displaystyle\iint_S f\,dS = \iint_D f(\mathbf{\Phi}(u,v))\,||\mathbf{T}_u\times\mathbf{T}_v||\,du\,dv$
2. Integral of a vector field $\mathbf{F}$: $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_D \mathbf{F}(\mathbf{\Phi}(u,v))\cdot(\mathbf{T}_u\times\mathbf{T}_v)\,du\,dv$

Green's Theorem: $\displaystyle\int_{\partial D} P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dx\,dy$

Special Case: $\displaystyle\frac{1}{2}\int_{\partial D} x\,dy - y\,dx = \iint_D dx\,dy$

Stoke's Theorem: $\displaystyle\iint_S (\boldsymbol{\nabla}\times\mathbf{F})\cdot d\mathbf{S} = \int_{\partial S}\mathbf{F}\cdot d\mathbf{s}$

Gauss' Theorem: $\displaystyle\iiint_W (\boldsymbol{\nabla}\cdot\mathbf{F})\,dV = \iint_{\partial W}\mathbf{F}\cdot d\mathbf{S}$

Math 20E Excel with Integrity Pledge

The Excel with Integrity pledge affirms the UC San Diego commitment to excel with integrity both on and off campus, in academic, professional, and research endeavors.

According to the International Center for Academic Integrity, academic integrity means having the courage to act in ways that are honest, fair, responsible, respectful & trustworthy even when it is difficult. Creating work with integrity is important because otherwise we are misrepresenting our knowledge and abilities and the University is falsely certifying our accomplishments. And when this happens, the UCSD degree loses its value and we've all wasted our time and talents!

Name: ______________________________ **PID:** ______________

Excel with Integrity Pledge

*I am **fair** to my classmates and instructors by not using any unauthorized aids.*
*I **respect** myself and my university by upholding educational and evaluative goals.*
*I am **honest** in my representation of myself and of my work.*
*I accept **responsibility** for ensuring my actions are in accord with academic integrity.*
*I show that I am **trustworthy** even when no one is watching.*

Affirm your adherence to this pledge by writing the following statement in the space below:

I Excel with Integrity.

Signature: ______________________________ **Date:** ____________