Math 20E Homework Assignment 2
Due 11:00pm Tuesday, April 23, 2024

1. Let D^* be the parallelogram with vertices at $(-1,3)$, $(0,0)$, $(2,-1)$, and $(1,2)$. Let D be the rectangle $D = [0,1] \times [0,1]$. Find a T such that D is the image set of D^* under T; that is, $D = T(D^*)$.

2. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the spherical coordinate mapping defined by $(\rho, \phi, \theta) \mapsto (x, y, z)$, where

$$x = \rho \sin(\phi)\cos(\theta), \qquad y = \rho \sin(\phi)\sin(\theta), \qquad z = \rho\cos(\phi).$$

Let D^* be the set of points (ρ, ϕ, θ) such that $\rho \in [0,1]$, $\phi \in [0,\pi]$, $\theta \in [0, 2\pi]$.

   (a) Find $D = T(D^*)$.

   (b) Is T one-to-one? If not, can we eliminate a subset $S \subseteq D^*$ so that T is one-to-one on the remainder $D^* \setminus S = \{(x,y,z) \in D^* \mid (x,y,z) \notin S\}$?

3. Evaluate $\iint_D x^2\, dx\, dy$ where D is determined by the two conditions $0 \le x \le y$ and $x^2 + y^2 \le 1$.

4. Evaluate $\iiint_W \sqrt{x^2+y^2+z^2}\, e^{-(x^2+y^2+z^2)}\, dx\, dy\, dz$, where W is the solid bounded by the two spheres
$x^2+y^2+z^2 = a^2$ and $x^2+y^2+z^2 = b^2$ with $0 < a < b$.

5. Evaluate $\iint_R (x+y)\, dx\, dy$, where R is the rectangle in the xy-plane with vertices at $(0,1), (1,0), (3,4), (4,3)$.

6. Show that the path $\mathbf{c}(t) = \big(\sin(t), \cos(t), e^t\big)$ is a flow line of the vector field $\mathbf{F}(x,y,z) = (y, -x, z)$.

7. Let $\mathbf{F}(x,y,z) = (yz, xz, xy)$. Find a function $f : \mathbb{R}^3 \to \mathbb{R}$ such that $\mathbf{F} = \nabla f$.

8. Evaluate the path integral $\int_{\mathbf{c}} f(x,y,z)\, ds$ with $f(x,y,z) = x+y+z$ and $\mathbf{c}(t) = (\sin(t), \cos(t), t)$ for $t \in [0, 2\pi]$.

9. Evaluate $\int_{\mathbf{c}} f\, ds$, where $f(x,y,z) = z$ and $\mathbf{c}(t) = (t\cos(t), t\sin(t), t)$ for $0 \le t \le t_0$.

10. Find the average z coordinate on the path $\mathbf{c}(t) = (t\cos(t), t\sin(t), t)$ for $0 \le t \le t_0$.