Math 20E
Formula Sheet

Vector Identities

1. $\nabla(f+g) = \nabla f + \nabla g$
2. $\nabla(cf) = c\,\nabla f$
3. $\nabla(fg) = f\,\nabla g + g\,\nabla f$
4. $\nabla\left(\frac{f}{g}\right) = \frac{g\,\nabla f - f\,\nabla g}{g^2}$, at points $\mathbf{x}$ where $g(\mathbf{x}) \neq 0$
5. $\nabla\cdot(\mathbf{F}+\mathbf{G}) = \nabla\cdot\mathbf{F} + \nabla\cdot\mathbf{G}$
6. $\nabla\times(\mathbf{F}+\mathbf{G}) = \nabla\times\mathbf{F} + \nabla\times\mathbf{G}$
7. $\nabla\cdot(f\mathbf{F}) = f\,\nabla\cdot\mathbf{F} + \mathbf{F}\cdot\nabla f$
8. $\nabla\times(f\mathbf{F}) = f\,\nabla\times\mathbf{F} + \nabla f\times\mathbf{F}$
9. $\nabla\cdot(\nabla\times\mathbf{F}) = 0$
10. $\nabla\times\nabla f = \mathbf{0}$
11. $\nabla\cdot(\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot(\nabla\times\mathbf{F}) - \mathbf{F}\cdot(\nabla\times\mathbf{G})$

Selected Integrals

1. $\int \tan(x)\,dx = \log|\sec(x)|$
2. $\int \sec(x)\,dx = \log|\sec(x)+\tan(x)|$
3. $\int \arcsin\left(\frac{x}{a}\right)\,dx = x\arcsin\left(\frac{x}{a}\right) + \sqrt{x^2+a^2}$
4. $\int \arctan\left(\frac{x}{a}\right)\,dx = x\arctan\left(\frac{x}{a}\right) - \frac{a}{2}\log(a^2+x^2)$
5. $\int \sin^2(x)\,dx = \frac{1}{2}\left(x - \sin(x)\cos(x)\right)$
6. $\int \cos^2(x)\,dx = \frac{1}{2}\left(x + \sin(x)\cos(x)\right)$
7. $\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \log(x+\sqrt{a^2+x^2})$
8. $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$
9. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\left(\frac{x}{a}\right)$
10. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right)$
11. $\int \frac{1}{\sqrt{x^2\pm a^2}}\,dx = \log(x+\sqrt{x^2\pm a^2})$
12. $\int \frac{1}{(a^2-x^2)^{3/2}}\,dx = \frac{x}{a^2\sqrt{a^2-x^2}}$
13. $\int \sqrt{x^2\pm a^2}\,dx = \frac{x}{2}\sqrt{x^2\pm a^2} \pm \frac{a^2}{2}\log(x+\sqrt{x^2\pm a^2})$
14. $\int (a^2-x^2)^{3/2}\,dx = \frac{x}{8}(5a^2-2x^2)\sqrt{a^2-x^2} + \frac{3a^4}{8}\arcsin\left(\frac{x}{a}\right)$

Path and Line Integrals over a Parameterized Curve (Path) $\mathbf{c}(t)$

1. Path integral of a scalar function f: $\displaystyle\int_{\mathbf{c}} f\,ds = \int_a^b f(\mathbf{c}(t))\,||\mathbf{c}'(t)||\,dt$
2. Line integral of a vector field $\mathbf{F}$: $\displaystyle\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{c}(t))\cdot\mathbf{c}'(t)\,dt$

Surface Integrals over a Parameterized Surface $\mathbf{\Phi}(u,v)$

1. Integral of a scalar function f: $\displaystyle\iint_S f\,dS = \iint_D f(\mathbf{\Phi}(u,v))\,||\mathbf{T}_u\times\mathbf{T}_v||\,du\,dv$
2. Integral of a vector field $\mathbf{F}$: $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_D \mathbf{F}(\mathbf{\Phi}(u,v))\cdot(\mathbf{T}_u\times\mathbf{T}_v)\,du\,dv$

Green's Theorem: $\displaystyle\int_{\partial D} P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dx\,dy$

Special Case: $\displaystyle\frac{1}{2}\int_{\partial D} x\,dy - y\,dx = \iint_D dx\,dy$

Stoke's Theorem: $\displaystyle\iint_S (\boldsymbol{\nabla}\times\mathbf{F})\cdot d\mathbf{S} = \int_{\partial S}\mathbf{F}\cdot d\mathbf{s}$

Gauss' Theorem: $\displaystyle\iiint_W (\boldsymbol{\nabla}\cdot\mathbf{F})\,dV = \iint_{\partial W}\mathbf{F}\cdot d\mathbf{S}$