Math 120A **Final Examination v. A** **Name:** ____________________

September 7, 2019 **(Total Points: *54*)** **PID:** ____________________

Instructions

1. **Write your Name and PID in the spaces provided above.**
2. **Make sure your Name is on every page.**
3. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
4. **Put away ANY devices that can be used for communication or can access the Internet.**
5. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
6. **Read each question carefully and answer each question completely.**
7. **Write your solutions clearly in the spaces provided. Work on scratch paper will not be graded.**
8. **Show all of your work. No credit will be given for unsupported answers, even if correct.**

(2 points) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

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(6 points) 1. Determine the value(s) of $\log\left[(1-i)^{i}\right]$.

(6 points) 2. Show that the series $\displaystyle\sum_{k=1}^{\infty} \frac{z^k}{k^2}$ converges uniformly for $|z| < 1$.

(6 points) 3. Find the two Laurent series expansions centered at $z = 0$ for $f(z) = \dfrac{1}{z - z^2}$ and determine the annulus of convergence for each of them.

(6 points) 4. $g(z) = e^{\frac{1}{z}}$ has an essential singularity at 0.

(a) Compute the residue of $g(z)$ at 0.

(b) Evaluate the integral $\oint_{|z|=2} g(z)\, dz$.

(6 points) 5. Suppose $f(z)$ is an entire function; that is, $f(z)$ is analytic on the entire complex plane $\mathbb{C}$. Suppose also that $f(z)$ is bounded; that is, there is $M > 0$ such that $|f(z)| \leq M$ for all $z \in \mathbb{C}$.

(a) Given any $z \in \mathbb{C}$, Cauchy's integral formula asserts that

$$f'(z) = \frac{1}{2\pi i} \int_{|\zeta - z| = R} \frac{f(\zeta)}{(\zeta - z)^2}\, d\zeta \quad \text{for every } R > 0.$$

Using an ML-estimate and letting $R \to \infty$, show that $|f'(z)| = 0$.

(b) Explain why this shows that any bounded entire function is constant.
[Remark: This amazing result is known as Liouville's theorem.]

(6 points) 6. Evaluate the improper integral

$$\int_0^\infty \frac{\cos(x)}{1+x^4}\,dx$$

as an appropriate limit of contour integrals, using the residue theorem. Be sure to clearly specify the contours you use and why the value of the integrals along the nonreal part of the contours tend to zero as $R \to \infty$.

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(6 points) 7. Evaluate the trigonometric integral

$$\int_0^{2\pi} \frac{\cos(\theta)}{5 - 4\cos(\theta)}\, d\theta$$

by representing it as a contour integral and using the residue theorem. Be sure to clearly specify the contour you use.

(10 points) 8. Answer each of the following multiple choice questions by circling the letter corresponding to your choice. Circle only one choice: Items with more than one choice circled will receive no credit, even if one of the choices is correct.

(These questions will be taken from the clicker questions you've seen during the term.)