Math 120A **Midterm Exam 2 v. A** **Name:** ____________

August 29, 2019 **(Total Points: *31*)** **PID:** ____________

Instructions

1. **Write your Name and PID in the spaces provided above.**
2. **Make sure your Name is on every page.**
3. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
4. **Put away ANY devices that can be used for communication or can access the Internet.**
5. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
6. **Read each question carefully and answer each question completely.**
7. **Write your solutions clearly in the spaces provided. Work on scratch paper will not be graded.**
8. **Show all of your work. No credit will be given for unsupported answers, even if correct.**

(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

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(6 points) 1. Given $R > 0$, let $\gamma = \{z \in \mathbb{C} \mid |z| = R\}$ with positive (counterclockwise) orientation. Evaluate the following integrals.

(a) $\displaystyle\int_\gamma |z^2| \, dz$

(b) $\displaystyle\int_\gamma |z^2| \, |dz|$

(6 points) 2. Let $\gamma = \{z \in \mathbb{C} \mid |z| = 1\}$ with positive (counterclockwise) orientation. Use the Cauchy integral formula to evaluate the following integrals.

(a) $\displaystyle\int_\gamma \frac{z^n}{z-2}\,dz$

(b) $\displaystyle\int_\gamma \frac{\sin(z)}{z}\,dz$

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(6 points) 3. Consider the function $f(z) = \dfrac{1}{z-1}$.

(a) Determine the power series expansion $\sum_{k=0}^{\infty} a_k \, (z-i)^k$ of $f(z)$ centered at $z_0 = i$.

(b) What is the radius of convergence of the power series?

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(6 points) 4. Let $f(z) = \begin{cases} 1 & \text{if } z = 0, \\ \frac{\sin(z)}{z} & \text{otherwise} \end{cases}.$

Compute the terms of the power series for $\dfrac{1}{f(z)} = \dfrac{z}{\sin(z)}$ up to order five.

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(6 points) 5. Let $g(z) = \dfrac{1}{z} + \dfrac{1}{z^5}$.

(a) Determine the zeros of g and their orders.

(b) Determine if g is analytic at ∞ and, if so, determine the order of the zero at ∞.