Math 120A **Midterm Exam 1 v. A** **Name:** ____________

August 15, 2019 **(Total Points: *31*)** **PID:** ____________

Instructions

1. **Write your Name and PID in the spaces provided above.**
2. **Make sure your Name is on every page.**
3. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
4. **Put away ANY devices that can be used for communication or can access the Internet.**
5. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
6. **Read each question carefully and answer each question completely.**
7. **Write your solutions clearly in the spaces provided. Work on scratch paper will not be graded.**
8. **Show all of your work. No credit will be given for unsupported answers, even if correct.**

(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Find the 4^{th} roots of $-i$. You may leave them in polar form.

(6 points) 2. Determine all values of $\left(\sqrt{3}+i\right)^{i}$.

(6 points) 3. Find the fractional linear transformation $g : \mathbb{C}^* \to \mathbb{C}^*$ that maps $(-i, -1, i)$ to $(0, 1, \infty)$.

(6 points) 4. Suppose $g : \mathbb{R}^2 \to \mathbb{R}^2$ given by $g(x, y) = (u(x, y),\, v(x, y))$ is differentiable with

$$\mathrm{Dg} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

(a) Explain how you know that g is analytic on $\mathbb{R}^2$.

(b) Given that $g(0, 0) = (0, 1)$, write an explicit formula for $g(x, y)$.

(c) Write $g : \mathbb{C} \to \mathbb{C}$ in complex form $g(z)$.

(6 points) 5. Let $D = \left\{(x, y) \mid x^2 + y^2 < 1\right\}$, the unit disk in $\mathbb{R}^2$. Evaluate the line integral

$$\int_{\partial D} xy^2\, dx + \left(x^2 y + 4x\right)\, dy$$

by using Green's theorem.