Math 120A
September 6, 2023

Question 1 Given $f(z) = \sum_{k=-\infty}^{\infty} b_k z^k$ for all $|z| > R$. $f(z)$ has a *removable* singularity at ∞ if

*A. $b_k = 0$ for all integers $k > 0$.

B. the principle part of $f(z)$ vanishes at ∞.

C. there is an integer $N \geq 1$ for which $b_N \neq 0$ but $b_k = 0$ for all integers $k > N$.

D. $b_k \neq 0$ for infinitely many integers $k > 0$.

E. none of the above; you can't remove singularities, especially at ∞.

Question 2 A function $f(z)$ has a nonzero residue at z_0. We can conclude that

A. z_0 is an isolated singularity of $f(z)$

B. the principal part of $f(z)$ is not zero.

C. z_0 is the only singularity of $f(z)$ in $|z - z_0| < \rho$ and $\displaystyle\int_{|\zeta - z_0| = \epsilon} f(\zeta)\, d\zeta \neq 0$ for every $0 < \epsilon < \rho$.

*D. all of the above.

E. none of the above.

Question 3 Suppose $f(z)$ has an essential singularity at z_0. Then,

A. $\text{Res}\,[f(z), z_0]$, the residue of $f(z)$ at z_0, is undefined.

B. $\displaystyle\int_{|\zeta - z_0| = \epsilon} f(\zeta)\, d\zeta$ is not defined for any $\epsilon > 0$.

C. $f(z)$ is not analytic at ∞.

D. all of the above.

*E. none of the above.

Question 4 Let $f(z) = \dfrac{1}{(z - z_0)^2}$. Then,

A. $\displaystyle\int_{|\zeta - z_0| = \epsilon} f(\zeta)\, d\zeta = 0$ for every $\epsilon > 0$.

B. $\displaystyle\int_{|\zeta - z_0| = \epsilon} f(\zeta)\, d\zeta = 2\pi i$ for every $\epsilon > 0$.

C. $\text{Res}[f(z), z_0] = 0$.

*D. **A** and **C**.

E. none of the above.

Question 5 Let $f(z) = \dfrac{1}{(z - z_0)}$. Then,

A. $\displaystyle\int_{|\zeta - z_0| = \epsilon} f(\zeta)\, d\zeta = 0$ for every $\epsilon > 0$.

*B. $\displaystyle\int_{|\zeta - z_0| = \epsilon} f(\zeta)\, d\zeta = 2\pi i$ for every $\epsilon > 0$.

C. $\text{Res}[f(z), z_0] = 0$.

D. **A** and **C**.

E. none of the above.