Math 120A
September 5, 2023

Question 1 Let z_0 be an isolated singularity of $f(z)$. Then,

*A. for some $r > 0$, $f(z)$ is analytic on $\{z \mid 0 < |z - z_0| < r\}$.

B. $\lim_{z \to z_0} |f(z)|$ diverges to $+\infty$.

C. $\frac{1}{2\pi i} \int_{|z-z_0|=r} \frac{f(z)}{z - z_0} = f(z_0)$ for some $r > 0$ by Cauchy's integral formula.

D. All of the above.

E. None of the above. Singularities can't be isolated.

Question 2 $\mathrm{Log}(z)$ is not analytic at $z_0 = 0$. $z_0 = 0$ is

A. an isolated singularity of $\mathrm{Log}(z)$.

*B. a branch point of $\mathrm{Log}(z)$.

C. an essential singularity of $\mathrm{Log}(z)$.

D. **A** and **B**

E. **A** and **C**

Question 3 $f(z)$ has a pole of order N at infinity if

A. the principal part of f at ∞ is a polynomial of degree N; that is, $P_\infty(z) = b_N z^N + b_{N-1} z^{N-1} + \cdots + b_1 z + b_0$.

B. $g(w) = f(1/w)$ has a pole of order N at $w = 0$.

C. $g(w) = f(1/w)$ has a zero of order N at $w = 0$.

*D. **A** and **B**

E. **A** and **C**

Question 4 Suppose z_0 is an isolated singularity of $f(z)$ so that $f(z)$ is analytic when $0 < |z - z_0| < \rho$. Then,

A. $f(z) = f_0(z) + f_1(z)$ with $f_0(z)$ analytic for $|z - z_0| < \rho$ and $f_1(z)$ analytic for $|z - z_0| > 0$.

B. If $0 < |z - z_0| < \rho$, $f(z)$ is represented by its Laurent series:
$$f(z) = \sum_{k=1}^{\infty} \frac{b_k}{(z - z_0)^k} + \sum_{k=0}^{\infty} a_k (z - z_0)^k.$$

C. $f_1(z) = \sum_{k=1}^{\infty} \frac{b_k}{(z - z_0)^k}$ is the principal part of $f(z)$ at z_0.

*D. All of the above.

E. None of the above. Functions can't be expanded around a singularity.

Question 5 What functions are analytic on the entire extended complex plane $\mathbb{C}^*$?

*A. constant functions $f(z) = c$

B. polynomials $f(z) = a_0 + a_1 z + \cdots + a_N z^N$

C. exponential functions $f(z) = a \exp(c\, z)$

D. **A** and **B**

E. all of the above