

Math 120A
August 28, 2023

Question 1 It is a foundational result of complex analysis that a function $f(z)$ analytic on a disk $|z - z_0| < R$ is represented by a power series $\sum_{k=0}^{\infty} a_k(z - z_0)^k$ with radius of convergence R . Why is this true?

- A. That's what *analytic* means. It's the definition.
- B. Real-valued functions with derivatives of all orders are analytic. It works the same for complex-valued functions.
- C. So that the studying complex analysis can be made a little bit simpler.
- *D. Cauchy's theorem allows us to write
$$f(z) = \sum_{k=0}^{\infty} \left(\frac{1}{2\pi i} \int_{|z-z_0|=r} \frac{f(w)}{(w - z_0)^{k+1}} dw \right) (z - z_0)^k$$
for each fixed $0 < r < R$.
- E. None of the above. What's a power series?

Question 2 The functions $f_k : [0, 1] \rightarrow \mathbb{R}$ given by $f_k(x) = x^k$

- A. are all continuous.
- B. converge pointwise to a discontinuous function.
- C. converge uniformly to a discontinuous function.
- *D. **A** and **B**.
- E. all of the above; if they converge uniformly, they also converge pointwise.

Question 3 Given a power series $f(z) = \sum_{k=0}^{\infty} a_k(z - z_0)^k$ with radius of convergence $R = +\infty$. We can conclude

- A. $f(z)$ converges for all $z \in \mathbb{C}$.
- B. $f(z)$ converges for all z with $|z - z_0| < R$.
- C. $f(z)$ converges for all z with $|z| < R$.
- *D. all of the above since every positive real number is less than $+\infty$.
- E. none of the above since $+\infty$ is not a positive real number and cannot be a radius of convergence.

Question 4 Suppose $\sum_{k=0}^{\infty} a_k z_0^k$ converges. We conclude that $\sum_{k=0}^{\infty} a_k z^k$

- A. converges absolutely for every z with $|z| < |z_0|$.
- B. converges uniformly for every z with $|z| \leq r$ whenever $r < |z_0|$.
- C. converges absolutely for every z with $|z| = |z_0|$.
- *D. **A** and **B**.
- E. all of the above.

Question 5 Given a power series $f(z) = \sum_{k=0}^{\infty} a_k(z - z_0)^k$ with radius of convergence $R = 0$. We can conclude

- A. $f(z)$ converges only when $z = z_0$.
- B. $f(z)$ converges for all z with $|z - z_0| \leq R$.
- C. $f(z)$ converges for all z with $|z - z_0| < R$.
- *D. **A** and **B**; they are the same.
- E. none of the above since a radius must be positive.