Math 120A
August 23, 2023

Question 1 Cauchy's integral formula states that a function $f(z)$ analytic on a bounded domain D and smooth on $D \cup \partial D$ satisfies

$$f(z) = \frac{1}{2\pi i} \int_{\partial D} \frac{f(w)}{z - w} dw, \quad z \in D.$$

This is important because

A. the integral can be differentiated with respect to w.

B. by induction, one sees that for every $z \in D$, $f^{(m)}(z) = \frac{m!}{2\pi i} \int_{\partial D} \frac{f(w)}{(z-w)^{m+1}} dw$ for all integers $m \geq 0$.

C. one can conclude that a function $f(z)$ that has one complex derivative at each $z \in D$ must have complex derivatives of all orders at each $z \in D$.

*D. All of the above, and it's absolutely amazing.

E. None of the above, Cauchy's integral formula is as useless as it is aesthetically pleasing.

Question 2 Let γ be the curve $|z| = 2$ with positive (counterclockwise) orientation. Then the integral $\int_\gamma \frac{z^n}{z-3}\,dz$

*A. is equal to 0 by Cauchy's theorem.

B. is equal to 3^n by the Cauchy integral theorem.

C. is equal to $2\pi i\, 3^n$ by the Cauchy integral theorem.

D. is undefined because $\frac{z^n}{z-3}$ is undefined at $z = 3$.

E. none of the above.

Question 3 Let γ be the curve $|z| = 2$ with positive (counterclockwise) orientation. Then the integral $\int_\gamma \frac{z^n}{z-1}\, dz$

A. is equal to 0 by Cauchy's theorem.

B. is equal to 1 by the Cauchy integral theorem.

*C. is equal to $2\pi i$ by the Cauchy integral theorem.

D. is undefined because $\frac{z^n}{z-1}$ is undefined at $z = 1$.

E. none of the above.

Question 4 Let γ be the curve $|z| = 2$ with positive (counterclockwise) orientation. Then the integral $\int_\gamma \frac{z^n}{z+2}\,dz$

A. is equal to 0 by Cauchy's theorem.

B. is equal to $(-2)^n$ by the Cauchy integral theorem.

C. is equal to $2\pi i\,(-2)^2$ by the Cauchy integral theorem.

*D. is undefined because $\frac{z^n}{z+2}$ is undefined at $z = -2$.

E. none of the above.

Question 5 Liouville's theorem asserts every bounded entire function is constant. Thus we can conclude that

A. if $f(z)$ is analytic on the complex plane $\mathbb{C}$ and there is a constant $M > 0$ for which $|f(z)| \leq M$ for every $z \in \mathbb{C}$, then $f'(z) = 0$ at every $z \in \mathbb{C}$.

B. if $p(z)$ is a nonconstant polynomial, then $p(z)$ has a complex zero; otherwise, $f(z) = \dfrac{1}{p(z)}$ would be a nonconstant bounded entire function.

C. the trigonometric functions $\cos(z)$ and $\sin(z)$ are unbounded since they are nonconstant entire functions.

D. **A** and **B**

*E. **A**, **B**, and **C**