Math 120A
August 22, 2023

**Question 1** Recall that the differential $-\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$ is defined on $\mathbb{C} \setminus \{0\}$ and has the following two properties:

1. $\frac{\partial}{\partial y}\left(-\frac{y}{x^2+y^2}\right) = \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right)$.
2. $\oint_{x^2+y^2=1} -\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy = 2\pi$.

Therefore, we can conclude that $-\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$

*A. is closed.

B. is exact.

C. is both closed and exact.

D. is neither closed nor exact.

E. violates Green's theorem.

**Question 2** A primitive of a continuous function $f : \mathbb{C} \to \mathbb{C}$ is

A. an antiderivative of $f$.

B. a function $F : \mathbb{C} \to \mathbb{C}$ such that $F'(z) = f(z)$.

C. an exact differential of $f$.

*D. both **A** and **B**.

E. all of the above.

**Question 3** Given a function $f(z)$ that is continuous on a domain $D$. If $F(z)$ is a primitive of $f(z)$ on $D$, then we can conclude that

A. $F'(z) = f(z)$ on $D$.

B. $F(w_2) - F(w_1) = \int_{w_1}^{w_2} f(z)\,dz$ along any path in $D$ from $w_1$ to $w_2$.

C. The differential $dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy$ is equal to $F'(z)\,dz$ and is exact on $D$.

*D. all of the above; they are equivalent.

E. none of the above; there is nothing primitive about complex functions.

**Question 4** Recall that $\text{Log}(z)$ is the principle branch of the logarithm and that $\text{Log}'(z) = \dfrac{1}{z}$ at all points $z \in \mathbb{C}$ where this makes sense. Thus,

A. $\text{Log}(z)$ is a primitive for $\dfrac{1}{z}$ on the punctured plane $\mathbb{C} \setminus \{0\}$ since neither $\text{Log}(z)$ nor $\dfrac{1}{z}$ are defined at 0.

B. $\text{Log}(z)$ is an antiderivative for $\dfrac{1}{z}$ on the slit plane $\mathbb{C} \setminus (-\infty, 0]$.

C. $\text{Log}(z)$ is a primitive for $\dfrac{1}{z}$ on the slit plane $\mathbb{C} \setminus (-\infty, 0]$.

*D. **B** and **C**; they are the same.

E. none of the above; slitting or puncturing planes is vandalism and is not allowed.

**Question 5** Let $\gamma : [a, b] \to \mathbb{C}$ be a piecewise smooth path with length $L$. We can conclude

A. $\left| \int_\gamma dz \right| \leq L$.

B. $\int_\gamma |dz| = L$.

C. $\int_a^b |\gamma'(t)|\, dt = L$.

D. **B** and **C**; they are the same.

*E. all of the above.