Math 120A
August 21, 2023

Question 1 A function is said to be *smooth* if it

    A. is continuous.

    B. is differentiable.

    C. is continuously differentiable.

    D. has derivatives of all orders (also called "infinitely differentiable").

    *E. has as many derivatives as necessary for whatever is being asserted to be true.

Question 2 Recall that the differential $-\dfrac{y}{x^2+y^2}dx + \dfrac{x}{x^2+y^2}dy$ is defined on $\mathbb{C} \setminus \{0\}$ and has the following two properties:

1. $\dfrac{\partial}{\partial y}\left(-\dfrac{y}{x^2+y^2}\right) = \dfrac{\partial}{\partial x}\left(\dfrac{x}{x^2+y^2}\right)$.

2. $\displaystyle\oint_{x^2+y^2=1} -\dfrac{y}{x^2+y^2}dx + \dfrac{x}{x^2+y^2}dy = 2\pi$.

Therefore, we can conclude that $-\dfrac{y}{x^2+y^2}dx + \dfrac{x}{x^2+y^2}dy$

   *A. is closed.

   B. is exact.

   C. is both closed and exact.

   D. is neither closed nor exact.

   E. violates Green's theorem.

Question 3 A continuous path $\gamma : [a, b] \to \mathbb{C}$ is *simple* if

- A. $\gamma(b) = \gamma(a)$.
- B. $\gamma(t_1) \neq \gamma(t_2)$ whenever $t_1 \neq t_2$.
- C. the image curve $\gamma([a, b])$ has no self-intersections.
- *D. **B** and **C**.
- E. all of the above.

Question 4 A continuous path $\gamma : [a, b] \to \mathbb{C}$ is *closed* if

- *A. $\gamma(b) = \gamma(a)$.
- B. $\gamma(t_1) \neq \gamma(t_2)$ whenever $t_1 \neq t_2$.
- C. the image curve $\gamma([a, b])$ has no self-intersections.
- D. **B** and **C**.
- E. all of the above.

Question 5 A set $D \subset \mathbb{C}$ is a *domain* if

    A. for every $z \in D$ there is $\epsilon > 0$ so that
       $\{w \in \mathbb{C} \mid |w - z| < \epsilon\} \subset D$.

    B. any two points in D can be connected by a continuous
       path consisting of a finite number of line segments.

    C. for every pair of points $z_1, z_2 \in D$, the line segment joining
       them is contained in D.

  *D. **A** and **B**.

    E. all of the above.