Math 120A
August 16, 2023

Question 1 A function $f(x, y) = (u(x, y), v(x, y))$ is complex differentiable at $z_0 = (x_0, y_0)$ if and only if

A. $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$ at (x_0, y_0).

B. $\dfrac{\partial}{\partial x}(u + iv) = \dfrac{1}{i}\dfrac{\partial}{\partial y}(u + iv)$ at (x_0, y_0).

C. $\lim\limits_{\Delta z \to 0} \dfrac{f(z_0 + \Delta z) - f(z)}{\Delta z}$ converges.

D. **A** and **C**.

*E. **A**, **B**, and **C**.

Question 2 Define $\log_1 : \mathbb{C} \setminus (-\infty, 0] \to \mathbb{C}$ by
$\log_1(z) = \log|z| + i\theta(z)$ with $-\pi < \theta < \pi$.

Define $\log_2 : \mathbb{C} \setminus [0, \infty) \to \mathbb{C}$ by
$\log_2(z) = \log|z| + i\phi(z)$ with $0 < \phi < 2\pi$.

Then,

A. $\log_1(z)$ and $\log_2(z)$ are analytic near $z = -i$.

B. $\log_1(-i) = \log_2(-i)$.

C. $\log_1'(-i) = \log_2'(-i)$.

D. **A** and **B**

*E. **A** and **C**

Question 3 $f : \mathbb{R}^2 \to \mathbb{R}^2$ has real derivative $\mathrm{Df} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$.

We can conclude that

A. f is real differentiable but not complex differentiable because $f'(z)$ cannot be written as a matrix.

B. f is complex differentiable because the partial derivatives of f satisfy the Cauchy-Riemann equations.

C. f is complex differentiable and $|f'(z)|^2 = \det(\mathrm{Df}) = 2$.

*D. **B** and **C**

E. We can't conclude anything. Not enough information has been provided.

Question 4 A fractional linear transformation is a function $f : C^* \to C^*$ of the form $f(z) = \dfrac{\alpha z + \beta}{\gamma z + \delta}$ with $\alpha\delta - \beta\gamma \neq 0$.

The reason for requiring that $\alpha\delta - \beta\gamma \neq 0$ is to ensure that

A. f is not constant

B. f has an inverse

C. $f'(z)$ is never zero

D. **A** and **C**; after all, they're the same

*E. **A**, **B**, and **C**; they're all great reasons

Question 5 Which of the following fractional linear transformations map $(-1, 0, 1) \mapsto (0, 1, \infty)$?

*A. $f(z) = \dfrac{z+1}{-z+1}$

B. $f(z) = \dfrac{z+1}{2z}$

C. $f(z) = \dfrac{2z-1}{z-1}$

D. $f(z) = \dfrac{z+1}{2z+1}$

E. None of the above.