Math 120A
August 14, 2023

Question 1 The power function z^α is single-valued

A. for every real number α.

B. for every rational number α.

*C. for every integer α.

D. All of the above; after all, every rational number is a real number and every integer is a rational number.

E. None of the above; z^α is always multiple-valued.

Question 2 Let $f(z) = e^z$ and $g(z) = z^{\frac{1}{4}}$.

A. $f(z)$ is single-valued, but $g(z)$ is multiple-valued.

B. $f\left(\frac{1}{4}\right) = g(e)$ since they are both equal to $e^{\frac{1}{4}}$.

C. $g(e) = \left\{e^{\frac{1}{4}+i\frac{\pi}{2}k},\ k = 0, 1, 2, 3\right\}$.

D. **B** and **C**

*E. **A** and **C**

Question 3 Let $f(z)$ and $g(z)$ be analytic for all $z \in \mathbb{C}$. Then,

A. $\dfrac{d}{dz}[f(z) + g(z)] = f'(z) + g'(z)$ (sum rule)

B. $\dfrac{d}{dz}[f(z)g(z)] = f'(z)g(z) + f(z)g'(z)$ (product rule)

C. $\dfrac{d}{dz} f(g(z)) = f'(g(z))\, g'(z)$ (chain rule)

*D. All of the above; these formulas work exactly the same as in real-variable calculus.

E. None of the above; the formulas only work in real-variable calculus where everything is single-valued.

Question 4 The hyperbolic functions $\cosh(z) = \frac{e^z+e^{-z}}{2}$ and $\sinh(z) = \frac{e^z-e^{-z}}{2}$ are

*A. periodic with period $2\pi i$, just like the complex exponential function e^z.

B. periodic with period 2π, just like the trigonometric functions $\cos(z)$ and $\sin(z)$.

C. not periodic; after all, they're hyperbolic.

D. never zero, just like the complex exponential function e^z.

E. **C** and **D**

Question 5 For every complex number z, the complex function $\gamma(z) = \bar{z}$ has the property that

A. $|\gamma(z)| = |z|$

B. $\gamma(z)$ is continuous at z.

C. $\gamma(z)$ is differentiable at z.

*D. **A** and **B**

E. **A**, **B** and **C**