Math 20E Homework Assignment 8
Due Friday, December 8, 2023 (not graded)

1. Let $\mathbf{r} = x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ and set $r = \|\mathbf{r}\| = \sqrt{x^2+y^2+z^2}$. Show that for a symmetric elementary region W with outward-oriented C^1 boundary surface ∂W,

$$\iiint_W \frac{1}{r^2}\,dV = \iint_{\partial W} \frac{\mathbf{r}}{r^2}\cdot \mathbf{n}\,dS.$$

Note: This is a simple application of Gauss's divergence theorem if $(0,0,0)\notin W$. Be sure to carefully show the equation still holds when $(0,0,0)\in W$.

2. Let $\mathbf{F}(x,y) = (xy,\ y^2)$.

   (a) Let $\mathbf{c}$ be the path $y = 2x^2$ in $\mathbb{R}^2$ joining $(0,0)$ and $(1,2)$. Evaluate $\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s}$.

   (b) Does the integral in part (a) depend on the choice of path joining $(0,0)$ to $(1,2)$?

3. Let $\mathbf{F}(x,y,z) = (e^x\sin(y))\,\mathbf{i} + (e^x\cos(y))\,\mathbf{j} + z^2\mathbf{k}$. Evaluate the integral $\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s}$, where $\mathbf{c}$ is the path given by $\mathbf{c}(t) = \left(\sqrt{t},\ t^3,\ \exp\left(\sqrt{t}\right)\right)$, with $0\le t\le 1$.

4. For each of the following vector fields $\mathbf{F}$, determine if it is a gradient field. If it is, find a function f such that $\nabla f = \mathbf{F}$.

   (a) $\mathbf{F}(x,y) = \left(2x+y^2-y\sin(x),\ 2xy+\cos(x)\right)$

   (b) $\mathbf{F}(x,y,z) = \left(6x^2z^2,\ 5x^2y^2,\ 4y^2z^2\right)$

   (c) $\mathbf{F}(x,y) = \left(y^3+1,\ 3xy^2+1\right)$

   (d) $\mathbf{F}(x,y) = \left(2x\cos(y), -x^2\sin(y)\right)$

5. Evaluate $\int_{\mathbf{c}} 2xyz\,dx + x^2z\,dy + x^26\,dz$ over the path $\mathbf{c}(t) = \left(t^2,\ \sin\left(\frac{\pi}{4}t\right),\ \exp\left(t^2-2t\right)\right)$ with $0\le t\le 2$.

6. Evaluate $\int_{\mathcal{C}} \sin(x)\,dx + z\cos(y)\,dy + \sin(y)\,dz$, where $\mathcal{C}$ is the ellipse defined by $4x^2+9y^2 = 36$ and oriented *clockwise*.