Math 20E Homework Assignment 6
Due Tuesday, November 21, 2022

1. Given any two positive constants c and d. Compute the area enclosed by the ellipse $\left(\frac{x}{c}\right)^2 + \left(\frac{y}{d}\right)^2 = 1.$

2. Find the area of the region between the x-axis and the cycloid parametrized by $\mathbf{r}(t) = (t - \sin(t),\ 1 - \cos(t))$ with $0 \le t \le 2\pi$.

3. The curve in $\mathbb{R}^2$ satisfying the equation $x^3 + y^3 = 3xy$ is called *the folium of Descartes.*
   (a) By setting $y = tx$, verify that $\mathbf{c} : \mathbb{R} \setminus \{-1\} \to \mathbb{R}^2$ with $\mathbf{c}(t) = \left(\frac{3t}{1+t^3}, \frac{3t^2}{1+t^3}\right)$ parametrizes the folium of Descartes. (Note: $\mathbb{R} \setminus \{-1\} = (-\infty, -1) \cup (-1, \infty)$.)
   (b) Use the parametrization above to determine the area enclosed by the loop of the folium. (Hint: The resulting integral should be an improper integral with $0 \le t < \infty$.)

4. Use Green's theorem to evaluate the line integral of $\mathbf{F} = \left(e^{x+y},\ e^{x-y}\right)$ along the clockwise-oriented curve consisting of the line segments joining the points $(0,0)$, $(2,2)$, $(4,2)$, $(2,0)$, and back to $(0,0)$ (and in that order). Be sure to note the orientation.

   (Hint: You might find the change of variables $u = x - y$, $v = y$ helpful when evaluating the double integral.)

5. Let $P(x,y) = \dfrac{-y}{x^2+y^2}$ and $Q(x,y) = \dfrac{x}{x^2+y^2}$, and let D be the unit disk $D = \{(x,y) \mid x^2+y^2 \le 1\}$.
   (a) Evaluate the area integral $\displaystyle\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\, dy$ over the unit disk D.
   (b) Evaluate the line integral $\displaystyle\int_{\partial D} P\, dx + Q\, dy$ around ∂D, the unit circle with positive orientation.
   (c) Briefly explain why Green's theorem failed.