Math 20E Homework Assignment 5
Due Tuesday, November 14, 2022

1. A metallic surface S is in the shape of a hemisphere $z = \sqrt{R^2 - x^2 - y^2}$, where (x, y) satisfies $x^2 + y^2 \leq R^2$. The mass density (mass per unit area) at $(x, y, z) \in S$ is given by $m(x, y, z) = x^2 + y^2$. Find the total mass of S.

2. Find the average value of $f(x, y, z) = x + z^2$ on the truncated cone $z^2 = x^2 + y^2$, with $3 \leq z \leq 4$.

3. Evaluate the integral $\iint_S (1-z)\, dS$, where S is the graph of $z = 1-x^2-y^2$, with $x^2+y^2 \leq 1$.

4. Find the mass of a spherical surface S of radius R with the property that at each point $(x, y, z) \in S$ the mass density is equal to a constant ρ times the distance between (x, y, z) and some fixed point $(x_0, y_0, z_0) \in S$.

5. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, with $\mathbf{F}(x, y, z) = (x, y, z)$, and S the part of the plane $x + y + z = 1$ with $x \geq 0$, $y \geq 0$, and $z \geq 0$.

6. Let $r = \sqrt{x^2 + y^2 + z^2}$ and let $\mathbf{e}_r = \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$ be the unit radial vector field. Compute the integral of $\mathbf{F} = e^{-r}\,\mathbf{e}_r$ over the upper hemisphere $x^2 + y^2 + z^2 = 9$ with outward-pointing normal.

7. Let $\mathcal{S}$ be the ellipsoid $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 + \left(\frac{z}{2}\right)^2 = 1$. Compute the flux of $\mathbf{F} = (0, 0, z)$ over the portion of $\mathcal{S}$ where $x \leq 0$, $y \leq 0$, $z \leq 0$ with upward-pointing normal.

8. Let $\mathbf{v} = (0, 0, z)$ be the velocity field (in meters per second) in $\mathbb{R}^3$. Compute the volume flow rate (in cubic meters per second) through the upper upper hemisphere $(z \geq 0)$ of the unit sphere $x^2 + y^2 + z^2 = 1$.

9. A net with surface described by $y = 0$ with $x^2 + z^2 \leq 1$ is dipped into a river in which the water flows according to the velocity field $\mathbf{v} = (x - y,\ z + y + 4,\ z^2)$. Determine the volume flow rate across the net.

10. Calculate the flux of the vector field $\mathbf{E}(x, y, z) = (0, 0, x)$ through the part of the ellipsoid

$$4x^2 + 9y^2 + z^2 = 36$$

where $z \geq 3$, $x \geq 0$, and $y \geq 0$.
(Hint: Use the parametrization $\Psi(r, \theta) = \left(3r\cos(\theta),\ 2r\sin(\theta),\ 6\sqrt{1 - r^2}\right)$ with an appropriate domain.)