Math 20E Homework Assignment 4
Due Tuesday, November 7, 2023

1. Evaluate $\displaystyle\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s}$, where $\mathbf{F}(x,y) = (-x\,y,\ x^2)$ and $\mathbf{c}$ is the path along the unit circle $x^2+y^2=1$ beginning at $(1,0)$ and ending at $(0,1)$.

2. Evaluate the line integral $\displaystyle\int_{\mathbf{c}} yz\,dx + xz\,dy + xy\,dz$, where $\mathbf{c}$ consists of the straight-line segments joining $(1,0,0)$ to $(0,1,0)$ to $(0,0,1)$.

3. Evaluate the line integral $\displaystyle\int_C (y^2+2xz)\,dx + (2xy+z^2)\,dy + (2yz+x^2)\,dz$, where C is an oriented simple curve from $(1,1,1)$ to $(0,2,3)$.

4. Let S be the surface parametrized by

$$x=\cos(u)\sin(v) \quad y=\sin(u)\sin(v) \quad z=\cos(v)$$

for $u\in[0,2\pi]$ and $v\in[0,\pi]$.

   (a) Find an expression for a unit vector normal to S at the image of a point $(u,v)\in[0,2\pi]\times[0,\pi]$.

   (b) Identify the surface S.

5. Let S be the surface determined by the equation $x^3+3xy+z^2=2$, with $z\geq 0$.

   (a) Find a parametrization $\Phi : D\subseteq\mathbb{R}^2\to S\subseteq\mathbb{R}^3$.

   (b) Find an equation for the tangent plane to S at the point $(1,1/3,0)$.

6. The image S of the parametrization

$$\Phi : [-\pi,\pi]\times[0,\pi]\to S\subseteq\mathbb{R}^3$$
$$\Phi(u,v) = (a\cos(u)\sin(v),\ b\sin(u)\sin(v),\ c\cos(v))$$

is an ellipsoid.

   (a) Find an equation for the ellipsoid S by evaluating the expression $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2+\left(\frac{z}{c}\right)^2$.

   (b) Show that Φ is not regular when $v=0$ or $v=\pi$.

   (c) Show that the image surface S is regular at all points of S.

7. Find the area of the unit sphere S parametrized by

$$\Phi : [0,2\pi]\times[0,\pi]\to S\subseteq\mathbb{R}^3$$
$$\Phi(\theta,\phi) = (\cos(\theta)\sin(\phi),\ \sin(\theta)\sin(\phi),\ \cos(\phi))$$

8. Find the area of the portion of the unit sphere that is inside the mouth of the cone $z\geq\sqrt{x^2+y^2}$.

9. The cylinder $x^2+y^2=x$ divides the unit sphere S into two regions S_1 and S_2, where S_1 is outside the cylinder and S_2 is inside the cylinder.

   Find the ratio $A(S_1)/A(S_2)$ of the areas of S_1 and S_2.

10. Find the area of the surface S defined by $x+y+z=1$, with $x^2+3y^2\leq 1$.