Math 20E Homework Assignment 3
Due Wednesday, October 24, 2023

1. Let D be the unit disk: $x^2 + y^2 \leq 1$. Evaluate $\iint_D \exp\left(x^2 + y^2\right)\, dx\, dy$.

2. Evaluate $\iint_D x^2\, dx\, dy$ where D is determined by the two conditions $0 \leq x \leq y$ and $x^2 + y^2 \leq 1$.

3. Evaluate $\iiint_W \sqrt{x^2 + y^2 + z^2}\, e^{-(x^2+y^2+z^2)}\, dx\, dy\, dz$, where W is the solid bounded by the two spheres
$x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$ with $0 < a < b$.

4. Evaluate $\iint_R (x+y)\, dx\, dy$, where R is the rectangle in the xy-plane with vertices at $(0,1), (1,0), (3,4), (4,3)$.

5. Show that the path $\mathbf{c}(t) = \left(\sin(t), \cos(t), e^t\right)$ is a flow line of the vector field $\mathbf{F}(x, y, z) = (y, -x, z)$.

6. Let $\mathbf{F}(x, y, z) = (yz, xz, xy)$. Find a function $f : \mathbb{R}^3 \to \mathbb{R}$ such that $\mathbf{F} = \nabla f$.

7. Evaluate the path integral $\int_{\mathbf{c}} f(x, y, z)\, ds$ with $f(x, y, z) = x + y + z$ and $\mathbf{c}(t) = (\sin(t), \cos(t), t)$ for $t \in [0, 2\pi]$.

8. Find the average y coordinate of the points on the semicircle parametrized by $\mathbf{c} : [0, \pi] \to \mathbb{R}^3$ given by $\mathbf{c}(t) = (0, a\sin(t), a\cos(t))$ with $a > 0$.

9. Evaluate $\int_{\mathbf{c}} f\, ds$, where $f(x, y, z) = z$ and $\mathbf{c}(t) = (t\cos(t), t\sin(t), t)$ for $0 \leq t \leq t_0$.

10. Find the average z coordinate on the path $\mathbf{c}(t) = (t\cos(t), t\sin(t), t)$ for $0 \leq t \leq t_0$.