Math 20E Homework Assignment 2
Due 11:00pm Tuesday, October 17, 2023

1. Change the order integration and evaluate:

$$\int_{y=0}^{1}\int_{x=y}^{1} \sin\left(x^2\right)\, dx\, dy.$$

2. Change the order integration and evaluate:

$$\int_{y=0}^{1}\int_{x=\sqrt{y}}^{1} e^{x^3}\, dx\, dy.$$

3. Let $D = [-1,1]\times[-1,2]$. Use the mean value inequality to show that

$$1 \le \iint_D \frac{1}{x^2+y^2+1}\, dx\, dy \le 6.$$

4. Compute $\iint_D f(x,y)\, dA$, where $f(x,y) = y^2\sqrt{x}$ and D is the set of (x,y) such that $x > 0$, $y > x^2$, and $y < 10 - x^2$.

5. Perform the indicated integration over the given box:

$$\iiint_B z\, e^{x+y}\, dx\, dy\, dz; \quad B = [0,1]\times[0,1]\times[0,1].$$

6. Find the volume of the solid bounded by $x^2 + 2y^2 = 2$, $z = 0$, and $x + y + 2z = 2$.

7. Evaluate the integral $\iiint_W z\, dx\, dy\, dz$; where W is the region bounded by $x = 0$, $y = 0$, $z = 0$, $z = 1$, and the cylinder $x^2 + y^2 = 1$, with $x \ge 0$, $y \ge 0$.

8. Let $S^* = (0,1]\times[0,2\pi)$ and define $T(r,\theta) = (r\cos(\theta),\ r\sin(\theta))$.
   (a) Determine the image set $S = T(S^*)$.
   (b) Show that T is one-to-one on S^*.

9. Let D^* be the parallelogram with vertices at $(-1,3)$, $(0,0)$, $(2,-1)$, and $(1,2)$. Let D be the rectangle $D = [0,1]\times[0,1]$. Find a T such that D is the image set of D^* under T; that is, $D = T(D^*)$.

10. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the spherical coordinate mapping defined by $(\rho,\phi,\theta) \mapsto (x,y,z)$, where

$$x = \rho\sin(\phi)\cos(\theta), \qquad y = \rho\sin(\phi)\sin(\theta), \qquad z = \rho\cos(\phi).$$

    Let D^* be the set of points (ρ,ϕ,θ) such that $\rho\in[0,1]$, $\phi\in[0,\pi]$, $\theta\in[0,2\pi]$.
    (a) Find $D = T(D^*)$.
    (b) Is T one-to-one? If not, can we eliminate a subset $S \subseteq D^*$ so that T is one-to-one on the remainder $D^*\setminus S = \{(x,y,z)\in D^* \mid (x,y,z)\notin S\}$?