

Math 120A
Midterm Exam 2
August 29, 2013
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Instructions

1. You may use any type of calculator, but no other electronic devices during this exam.
 - Express numbers symbolically (for example, $\sqrt{2}$ rather than 2.1).
2. You may use one page of notes, but no books or other assistance during this exam.
3. Write your *Name*, *PID*, and *Section* on the front of your Blue Book.
4. Write your solutions clearly in your Blue Book
 - (a) Carefully indicate the number and letter of each question and question part.
 - (b) Present your answers in the same order they appear in the exam.
 - (c) Start each question on a new side of a page.
5. Read each question carefully, and answer each question completely.
6. Show all of your work; no credit will be given for unsupported answers.

1. (6 points) Compute $\sin^{-1}\left(-\frac{5}{4}\right)$ using the principal branch of the logarithm and the branch $\sqrt{r}e^{i\frac{\theta}{2}}$ with $0 < \theta < 2\pi$ of the square root. Use the formula $\sin^{-1}(z) = -i \log \left[iz + \sqrt{1 - z^2} \right]$.
2. (6 points) Consider the function $f(z) = \sin(\bar{z})$.
 - (a) At what point(s) is $f(z)$ differentiable?
 - (b) At what point(s) is $f(z)$ analytic?
3. (6 points) Evaluate $\oint_C \frac{2z}{z^2 + 1} dz$, where C is the circle $|z| = 2$. You may use the fact that $\frac{2z}{z^2 + 1} = \frac{1}{z - i} + \frac{1}{z + i}$.
4. (6 points) Determine an upper bound for $\left| \oint_C \frac{z^3}{(z^2 + 4)^2} dz \right|$, where C is the circle $|z| = 3$.
5. (6 points) Find the radius of convergence of the power series $\sum_{k=1}^{\infty} \frac{1}{k} \left(\frac{i}{i+1} \right) \frac{z^k}{3^k}$.