Math 120A
September 1, 2022

Question 1 Suppose $f(z)$ has an essential singularity at z_0. Then,

A. $\text{Res}\,[f(z), z_0]$, the residue of $f(z)$ at z_0, is undefined.

B. $\displaystyle\int_{|\zeta - z_0| = \epsilon} f(\zeta)\, d\zeta$ is not defined for any $\epsilon > 0$.

C. $f(z)$ is not analytic at ∞.

*D. $\text{Res}\,[f(z), z_0]$ is the coefficient of $(z - z_0)^{-1}$ in the Laurent series for f centered at z_0.

E. none of the above.

Question 2 A function $f(z)$ has a nonzero residue at z_0 if

A. z_0 is an isolated singularity of $f(z)$

B. the principal part of $f(z)$ is not zero.

*C. z_0 is the only singularity of $f(z)$ in $|z - z_0| < \rho$ and $\int_{|\zeta - z_0| = \epsilon} f(\zeta)\, d\zeta \neq 0$ for every $0 < \epsilon < \rho$.

D. all of the above.

E. none of the above.

Question 3 It is a foundational result of complex analysis that a function $f(z)$ analytic on a disk $|z - z_0| < R$ is represented by a power series $\sum_{k=0}^{\infty} a_k (z - z_0)^k$ with radius of convergence R. Why is this true?

A. That's what *analytic* means. It's the definition.

B. Real-valued functions with derivatives of all orders are analytic. It works the same for complex-valued functions.

C. So that the studying complex analysis can be made a little bit simpler.

*D. Cauchy's theorem allows us to write

$$f(z) = \sum_{k=0}^{\infty} \left(\frac{1}{2\pi i} \int_{|z-z_0|=r} \frac{f(w)}{(w - z_0)^{k+1}} dw \right) z^k$$

for each fixed $0 < r < R$.

E. None of the above. What's a power series?

Question 4 Let $f(z) = e^z$ and $g(z) = z^{\frac{1}{4}}$.

A. $f(z)$ is single-valued, but $g(z)$ is multiple-valued.

B. $f\left(\frac{1}{4}\right) = g(e)$ since they are both equal to $e^{\frac{1}{4}}$.

C. $g(e) = \left\{e^{\frac{1}{4}+i\frac{\pi}{2}k},\ k = 0, 1, 2, 3\right\}$.

D. **B** and **C**

*E. **A** and **C**

Question 5 Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x, y) = (u(x, y), v(x, y))$. Suppose f has continuous partial derivatives. Then,

A. f is differentiable.

B. $\mathrm{Df} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}$ is the derivative of f.

C. Viewing f as $f(x + iy) = u(x + iy) + iv(x + iy)$, f is complex differentiable.

*D. **A** and **B**

E. **A**, **B**, and **C**

Question 6 A function $f(x, y) = (u(x, y), v(x, y))$ is complex differentiable at $z_0 = (x_0, y_0)$ if and only if

A. $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$ at (x_0, y_0).

B. $\dfrac{\partial}{\partial x}(u + iv) = \dfrac{1}{i}\dfrac{\partial}{\partial y}(u + iv)$ at (x_0, y_0).

C. $\lim\limits_{\Delta z \to 0} \dfrac{f(z_0 + \Delta z) - f(z)}{\Delta z}$ converges.

D. **A** and **C**.

*E. **A**, **B**, and **C**.

Question 7 Suppose $\sum_{k=0}^{\infty} a_k z_0^k$ converges. We conclude that $\sum_{k=0}^{\infty} a_k z^k$

A. converges absolutely for every z with $|z| < |z_0|$.

B. converges uniformly for every z with $|z| \leq r$ whenever $r < |z_0|$.

C. converges absolutely for every z with $|z| = |z_0|$.

*D. **A** and **B**.

E. all of the above.

Question 8 $f(z)$ has a pole of order N at infinity if

A. $P_\infty(z)$, the principal part of $f(z)$ at ∞, is a polynomial of degree N; i.e., $P_\infty(z) = b_N z^N + b_{N-1} z^{N-1} + \cdots b_1 z + b_0$.

B. $g(w) = f(1/w)$ has a pole of order N at $w = 0$.

C. $g(w) = f(1/w)$ has a zero of order N at $w = 0$.

*D. **A** and **B**

E. **A** and **C**