Math 120A
August 30, 2022

Question 1 $f(z)$ has a pole of order N at infinity if

  A. $P_\infty(z)$, the principal part of $f(z)$ at ∞, is a polynomial of degree N; i.e., $P_\infty(z) = b_N z^N + b_{N-1} z^{N-1} + \cdots b_1 z + b_0$.

  B. $g(w) = f(1/w)$ has a pole of order N at $w = 0$.

  C. $g(w) = f(1/w)$ has a zero of order N at $w = 0$.

  *D. **A** and **B**

  E. **A** and **C**

Question 2 z_0 is an isolated singularity of $f(z)$ so that $f(z)$ is analytic when $0 < |z - z_0| < \rho$.

- A. $f(z) = f_0(z) + f_1(z)$ with $f_0(z)$ analytic for $|z - z_0| < \rho$ and $f_1(z)$ analytic for $|z - z_0| > 0$.

- B. If $0 < |z - z_0| < \rho$, $f(z)$ is represented by its Laurent series:
  $f(z) = \sum_{k=1}^{\infty} \frac{b_k}{(z-z_0)^k} + \sum_{k=0}^{\infty} a_k (z - z_0)^k$.

- C. $f_1(z) = \sum_{k=1}^{\infty} \frac{b_k}{(z-z_0)^k}$ is the principal part of $f(z)$ at z_0.

- *D. All of the above.

- E. None of the above. Functions can't be expanded around a singularity.

Question 3 Let $f(z) = \dfrac{1}{1+z^2}$.

A. $f(z) = \displaystyle\sum_{k=0}^{\infty}(-1)^k z^{2k}$ is the power series for f centered at 0 and converges for $|z| < 1$.

B. $f(z) = \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{z^{2k}}$ is the power series for f centered at ∞ and converges for $|z| > 1$.

C. $g(w) = f(1/w) = \frac{w^2}{w^2+1}$, so $f(z)$ has a double zero at ∞.

D. **A** and **C**.

*E. All of the above.

Question 4 Let z_0 be an isolated singularity of $f(z)$. Then,

*A. for some $r > 0$, $f(z)$ is analytic on $\{z \mid 0 < |z| < r\}$.

B. $\lim\limits_{z \to z_0} |f(z)|$ diverges to $+\infty$.

C. $\dfrac{1}{2\pi i} \displaystyle\int_{|z|=r} \dfrac{f(z)}{z - z_0} = f(z_0)$ for some $r > 0$ by Cauchy's integral formula.

D. All of the above.

E. None of the above. Singularities can't be isolated.

Question 5 $\text{Log}(z)$ is not analytic at $z_0 = 0$. $z_0 = 0$ is

    A. an isolated singularity of $\text{Log}(z)$

    *B. a branch point of $\text{Log}(z)$

    C. an essential singularity of $\text{Log}(z)$

    D. **A** and **B**

    E. **A** and **C**