Math 120A
August 24, 2022

Question 1 Given a power series $f(z) = \sum_{k=0}^{\infty} a_k (z - z_0)^k$ with radius of convergence $R = +\infty$. We can conclude

A. $f(z)$ converges for all $z \in \mathbb{C}$.

B. $f(z)$ converges for all z with $|z - z_0| < R$.

C. $f(z)$ converges for all z with $|z| < R$.

*D. all of the above since every positive real number is less than $+\infty$.

E. none of the above since $+\infty$ is not a positive real number and cannot be a radius of convergence.

Question 2 Suppose $\sum_{k=0}^{\infty} a_k z_0^k$ converges. We conclude that $\sum_{k=0}^{\infty} a_k z^k$

A. converges absolutely for every z with $|z| < |z_0|$.

B. converges uniformly for every z with $|z| \leq r$ whenever $r < |z_0|$.

C. converges absolutely for every z with $|z| = |z_0|$.

*D. **A** and **B**.

E. all of the above.

Question 3 An analytic function $f(z)$ has a zero of order N at z_0 if

A. $f(z) = h(z)(z - z_0)^N$ for some analytic function $h(z)$ with $h(z_0) \neq 0$

B. $f(z_0) = f'(z_0) = \cdots = f^{(N-1)}(z_0) = 0$ and $f^{(N)}(z_0) \neq 0$

C. $f(z) = g(z)^N$ for some analytic function $g(z)$ with $g'(z_0) \neq 0$

*D. All of the above

E. None of the above. Zeros don't have any order

Question 4 Let $f(z) = \dfrac{1}{1+z^2}$.

A. $f(z) = \sum_{k=0}^{\infty} (-1)^k z^{2k}$ is the power series for f centered at 0 and converges for $|z| < 1$.

B. $f(z) = \sum_{k=1}^{\infty} \dfrac{(-1)^{k+1}}{z^{2k}}$ is the power series for f centered at ∞ and converges for $|z| > 1$.

C. $f(z)$ has a zero at ∞.

D. **A** and **C**.

*E. All of the above.