Math 120A
August 23, 2022

Question 1 Given a power series $f(z) = \sum_{k=0}^{\infty} a_k(z - z_0)^k$ with radius of convergence $R = +\infty$. We can conclude

A. $f(z)$ converges for all $z \in \mathbb{C}$.

B. $f(z)$ converges for all z with $|z - z_0| < R$.

C. $f(z)$ converges for all z with $|z| < R$.

*D. all of the above since every positive real number is less than $+\infty$.

E. none of the above since $+\infty$ is not a positive real number and cannot be a radius of convergence.

Question 2 Suppose $\sum_{k=0}^{\infty} a_k z_0^k$ converges. We conclude that $\sum_{k=0}^{\infty} a_k z^k$

A. converges absolutely for every z with $|z| < |z_0|$.

B. converges uniformly for every z with $|z| \leq r$ whenever $r < |z_0|$.

C. converges absolutely for every z with $|z| = |z_0|$.

*D. **A** and **B**.

E. all of the above.

Question 3 Given a power series $f(z) = \sum_{k=0}^{\infty} a_k (z - z_0)^k$ with radius of convergence $R = 0$. We can conclude

A. $f(z)$ converges only when $z = z_0$.

B. $f(z)$ converges for all z with $|z - z_0| \leq R$.

C. $f(z)$ converges for all z with $|z - z_0| < R$.

*D. **A** and **B**; they are the same.

E. none of the above since a radius must be positive.

Question 4 It is a foundational result of complex analysis that a function $f(z)$ analytic on a disk $|z - z_0| < R$ is represented by a power series $\sum_{k=0}^{\infty} a_k(z - z_0)^k$ with radius of convergence R. Why is this true?

A. That's what *analytic* means. It's the definition.

B. Real-valued functions with derivatives of all orders are analytic. It works the same for complex-valued functions.

C. So that the studying complex analysis can be made a little bit simpler.

*D. Cauchy's theorem allows us to write

$$f(z) = \sum_{k=0}^{\infty} \left(\frac{1}{2\pi i} \int_{|z-z_0|=r} \frac{f(w)}{(w - z_0)^{k+1}} dw \right) z^k$$

for each fixed $0 < r < R$.

E. None of the above. What's a power series?