Math 120A
August 16, 2022

Question 1 A function is said to be *smooth* if it

A. is continuous.

B. is differentiable.

C. is continuously differentiable.

D. has derivatives of all orders (also called "infinitely differentiable").

*E. has as many derivatives as necessary for whatever is being asserted to be true.

Question 2 Recall that the differential $-\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$ has the following two properties:

1. $\frac{\partial}{\partial y}\left(-\frac{y}{x^2+y^2}\right) = \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right)$.
2. $\oint_{x^2+y^2=1} -\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy = 2\pi$.

Therefore, we can conclude that $-\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$

*A. is closed.

B. is exact.

C. is both closed and exact.

D. is neither closed nor exact.

E. violates Green's theorem.

Question 3 A primitive of a continuous function $f : \mathbb{C} \to \mathbb{C}$ is

A. an antiderivative of f.

B. a function $F : \mathbb{C} \to \mathbb{C}$ such that $F'(z) = f(z)$.

C. an exact differential of f.

*D. both **A** and **B**.

E. all of the above.

Question 4 Recall that $\mathrm{Log}(z)$ is the principle branch of the logarithm and that $\mathrm{Log}'(z) = \frac{1}{z}$ at all points $z \in \mathbb{C}$ where this makes sense. Thus,

A. $\mathrm{Log}(z)$ is a primitive for $\frac{1}{z}$ on the punctured plane $\mathbb{C} \setminus \{0\}$ since neither $\mathrm{Log}(z)$ nor $\frac{1}{z}$ are defined at 0.

B. $\mathrm{Log}(z)$ is an antiderivative for $\frac{1}{z}$ on the slit plane $\mathbb{C} \setminus (-\infty, 0]$.

C. $\mathrm{Log}(z)$ is a primitive for $\frac{1}{z}$ on the slit plane $\mathbb{C} \setminus (-\infty, 0]$.

*D. **B** and **C**; they are the same.

E. none of the above; slitting or puncturing planes is vandalism and is not allowed.

Question 5 A continuous path $\gamma : [a, b] \to \mathbb{C}$ is *simple* if

A. $\gamma(b) = \gamma(a)$.

*B. $\gamma(t_1) \neq \gamma(t_2)$ whenever $t_1 \neq t_2$.

C. the image curve $\gamma([a, b])$ has no self-intersections.

D. **B** and **C**.

E. all of the above.

Question 6 A continuous path $\gamma : [a, b] \to \mathbb{C}$ is *closed* if

*A. $\gamma(b) = \gamma(a)$.

B. $\gamma(t_1) \neq \gamma(t_2)$ whenever $t_1 \neq t_2$.

C. the image curve $\gamma([a, b])$ has no self-intersections.

D. **B** and **C**.

E. all of the above.