Math 120A
August 10, 2022

Question 1 A function $f(x,y) = (u(x,y), v(x,y))$ is complex differentiable at $z_0 = (x_0, y_0)$ if and only if

A. $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$ at (x_0, y_0).

B. $\dfrac{\partial}{\partial x}(u + iv) = \dfrac{1}{i}\dfrac{\partial}{\partial y}(u + iv)$ at (x_0, y_0).

C. $\lim\limits_{\Delta z \to 0} \dfrac{f(z_0 + \Delta z) - f(z)}{\Delta z}$ converges.

D. **A** and **C**.

*E. **A**, **B**, and **C**.

Question 2 Let $f(z) = e^z = \exp(z)$ and $g(z) = z^{\frac{1}{4}}$.

A. $f(z)$ is single-valued, but $g(z)$ is multiple-valued.

B. $f\left(\frac{1}{4}\right) = g(e)$ since they are both equal to $e^{\frac{1}{4}}$.

C. $g(e) = \left\{e^{\frac{1}{4}+i\frac{\pi}{2}k},\ k = 0, 1, 2, 3\right\}$.

D. **B** and **C**

*E. **A** and **C**

Question 3 Define $\log_1 : \mathbb{C} \setminus (-\infty, 0] \to \mathbb{C}$ by
$\log_1(z) = \log|z| + i\theta(z)$ with $-\pi < \theta < \pi$.
Define $\log_2 : \mathbb{C} \setminus [0, \infty) \to \mathbb{C}$ by
$\log_2(z) = \log|z| + i\phi(z)$ with $0 < \phi < 2\pi$.
Then,

A. $\log_1(z)$ and $\log_2(z)$ are analytic near $z = -i$.

B. $\log_1(-i) = \log_2(-i)$.

C. $\log_1'(-i) = \log_2'(-i)$.

D. **A** and **B**

*E. **A** and **C**

Question 4 Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x, y) = (u(x, y), v(x, y))$. Suppose f has continuous partial derivatives. Then,

A. f is differentiable.

B. $\text{Df} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}$ is the derivative of f.

C. Viewing f as $f(x + iy) = u(x + iy) + iv(x + iy)$, f is complex differentiable.

*D. **A** and **B**

E. **A**, **B**, and **C**

Question 5 $f : \mathbb{R}^2 \to \mathbb{R}^2$ has derivative $\text{Df} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$.

We can conclude that

A. f is differentiable but not complex differentiable because $f'(z)$ cannot be written as a matrix.

B. f is complex differentiable because the partial derivatives of f satisfy the Cauchy-Riemann equations.

C. f is complex differentiable and $|f'(z)|^2 = \det(\text{Df}) = 2$.

*D. **B** and **C**

E. We can't conclude anything. Not enough information has been provided.