Math 120A
August 8, 2022

Question 1 The power function z^α is single-valued

A. for every real number α.

B. for every rational number α.

*C. for every integer α.

D. All of the above; after all, every rational number is a real number and every integer is a rational number.

E. None of the above; z^α is always multiple-valued.

Question 2 Let $f(z) = e^z$ and $g(z) = z^{\frac{1}{4}}$.

A. $f(z)$ is single-valued, but $g(z)$ is multiple-valued.

B. $f\left(\frac{1}{4}\right) = g(e)$ since they are both equal to $e^{\frac{1}{4}}$.

C. $g(e) = \left\{e^{\frac{1}{4}+i\frac{\pi}{2}k},\ k = 0, 1, 2, 3\right\}$.

D. **B** and **C**

*E. **A** and **C**

Question 3 Let $f(z)$ and $g(z)$ be analytic for all $z \in \mathbb{C}$. Then,

A. $\frac{d}{dz}[f(z)+g(z)] = f'(z)+g'(z)$ (sum rule)

B. $\frac{d}{dz}[f(z)g(z)] = f'(z)g(z)+f(z)g'(z)$ (product rule)

C. $\frac{d}{dz}f(g(z)) = f'(g(z))\,g'(z)$ (chain rule)

*D. All of the above; these formulas work exactly the same as in real-variable calculus.

E. None of the above; the formulas only work in real-variable calculus where everything is single-valued.

Question 4 A function $f(x, y) = (u(x, y), v(x, y))$ is complex differentiable at $z_0 = (x_0, y_0)$ if and only if

A. $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$ at (x_0, y_0).

B. $\dfrac{\partial}{\partial x}(u + iv) = \dfrac{1}{i}\dfrac{\partial}{\partial y}(u + iv)$ at (x_0, y_0).

C. $\lim\limits_{\Delta z \to 0} \dfrac{f(z_0 + \Delta z) - f(z)}{\Delta z}$ converges.

D. **A** and **C**.

*E. **A**, **B**, and **C**.

Question 5 Suppose $f(z) = u(x, y) + iv(x, y)$ is analytic on a domain D. Suppose further that $f(z)$ is real-valued on D. Then,

A. $v = 0$ on D.

B. $\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} = 0$ on D.

C. $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = 0$ on D.

*D. All of the above; in fact, f is constant on D.

E. None of the above. There are no real-valued analytic functions.