Math 20E Homework Assignment 4
Due Monday, October 31, 2022

1. Evaluate $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{F}(x, y) = (-x\,y,\ x^2)$ and $\mathbf{c}$ is the path along the unit circle $x^2 + y^2 = 1$ beginning at $(1, 0)$ and ending at $(0, 1)$.

2. Evaluate the line integral $\int_{\mathbf{c}} yz\,dx + xz\,dy + xy\,dz$, where $\mathbf{c}$ consists of the straight-line segments joining $(1, 0, 0)$ to $(0, 1, 0)$ to $(0, 0, 1)$.

3. Evaluate the line integral $\int_C (y^2 + 2xz)\,dx + (2xy + z^2)\,dy + (2yz + x^2)\,dz$, where C is an oriented simple curve from $(1, 1, 1)$ to $(0, 2, 3)$.

4. Let S be the surface parametrized by

$$x = \cos(u)\sin(v) \quad y = \sin(u)\sin(v) \quad z = \cos(v)$$

   for $u \in [0, 2\pi]$ and $v \in [0, \pi]$.

   (a) Find an expression for a unit vector normal to S at the image of a point $(u, v) \in [0, 2\pi] \times [0, \pi]$.

   (b) Identify the surface S.

5. Let S be the surface determined by the equation $x^3 + 3xy + z^2 = 2$, with $z \geq 0$.

   (a) Find a parametrization $\Phi : D \subseteq \mathbb{R}^2 \to S \subseteq \mathbb{R}^3$.

   (b) Find an equation for the tangent plane to S at the point $(1, 1/3, 0)$.

6. The image S of the parametrization

$$\Phi : [-\pi, \pi] \times [0, \pi] \to S \subseteq \mathbb{R}^3$$
$$\Phi(u, v) = (a\cos(u)\sin(v),\ b\sin(u)\sin(v),\ c\cos(v))$$

   is an ellipsoid.

   (a) Find an equation for the ellipsoid S by evaluating the expression $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2$.

   (b) Show that Φ is not regular when $v = 0$ or $v = \pi$.

   (c) Show that the image surface S is regular at all points of S.

7. Find the area of the unit sphere S parametrized by

$$\Phi : [0, 2\pi] \times [0, \pi] \to S \subseteq \mathbb{R}^3$$
$$\Phi(\theta, \phi) = (\cos(\theta)\sin(\phi),\ \sin(\theta)\sin(\phi),\ \cos(\phi))$$

8. Find the area of the portion of the unit sphere that is inside the mouth of the cone $z \geq \sqrt{x^2 + y^2}$.

9. The cylinder $x^2 + y^2 = x$ divides the unit sphere S into two regions S_1 and S_2, where S_1 is outside the cylinder and S_2 is inside the cylinder.

   Find the ratio $A(S_1)/A(S_2)$ of the areas of S_1 and S_2.

10. Find the area of the surface S defined by $x + y + z = 1$, with $x^2 + 3y^2 \leq 1$.