

Math 20E Homework Assignment 3
Due Monday, October 17, 2022

1. Let D be the unit disk: $x^2 + y^2 \leq 1$. Evaluate $\iint_D \exp(x^2 + y^2) dx dy$.
2. Evaluate $\iint_D x^2 dx dy$ where D is determined by the two conditions $0 \leq x \leq y$ and $x^2 + y^2 \leq 1$.
3. Evaluate $\iiint_W \sqrt{x^2 + y^2 + z^2} e^{-(x^2 + y^2 + z^2)} dx dy dz$, where W is the solid bounded by the two spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$ with $0 < a < b$.
4. Evaluate $\iint_R (x+y) dx dy$, where R is the rectangle in the xy -plane with vertices at $(0, 1), (1, 0), (3, 4), (4, 3)$.
5. Show that the path $\mathbf{c}(t) = (\sin(t), \cos(t), e^t)$ is a flow line of the vector field $\mathbf{F}(x, y, z) = (y, -x, z)$.
6. Let $\mathbf{F}(x, y, z) = (yz, xz, xy)$. Find a function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ such that $\mathbf{F} = \nabla f$.
7. Evaluate the path integral $\int_{\mathbf{c}} f(x, y, z) ds$ with $f(x, y, z) = x + y + z$ and $\mathbf{c}(t) = (\sin(t), \cos(t), t)$ for $t \in [0, 2\pi]$.
8. Find the average y coordinate of the points on the semicircle parametrized by $\mathbf{c} : [0, \pi] \rightarrow \mathbb{R}^3$ given by $\mathbf{c}(t) = (0, a \sin(t), a \cos(t))$ with $a > 0$.
9. Evaluate $\int_{\mathbf{c}} f ds$, where $f(x, y, z) = z$ and $\mathbf{c}(t) = (t \cos(t), t \sin(t), t)$ for $0 \leq t \leq t_0$.
10. Find the average z coordinate on the path $\mathbf{c}(t) = (t \cos(t), t \sin(t), t)$ for $0 \leq t \leq t_0$.