**Math 20E Homework Assignment 2**
**Due Monday, October 10, 2022**

1. Change the order integration and evaluate:

$$\int_{y=0}^{1} \int_{x=y}^{1} \sin\left(x^2\right)\, dx\, dy.$$

2. Change the order integration and evaluate:

$$\int_{y=0}^{1} \int_{x=\sqrt{y}}^{1} e^{x^3}\, dx\, dy.$$

3. Let $D = [-1, 1] \times [-1, 2]$. Use the mean value inequality to show that

$$1 \le \iint_D \frac{1}{x^2 + y^2 + 1}\, dx\, dy \le 6.$$

4. Compute $\iint_D f(x, y)\, dA$, where $f(x, y) = y^2\sqrt{x}$ and $D$ is the set of $(x, y)$ such that $x > 0$, $y > x^2$, and $y < 10 - x^2$.

5. Perform the indicated integration over the given box:

$$\iiint_B z\, e^{x+y}\, dx\, dy\, dz; \quad B = [0, 1] \times [0, 1] \times [0, 1].$$

6. Find the volume of the solid bounded by $x^2 + 2y^2 = 2$, $z = 0$, and $x + y + 2z = 2$.

7. Evaluate the integral $\iiint_W z\, dx\, dy\, dz$; where $W$ is the region bounded by $x = 0$, $y = 0$, $z = 0$, $z = 1$, and the cylinder $x^2 + y^2 = 1$, with $x \ge 0$, $y \ge 0$.

8. Let $S^* = (0, 1] \times [0, 2\pi)$ and define $T(r, \theta) = (r\cos(\theta),\ r\sin(\theta))$.
   (a) Determine the image set $S = T(S^*)$.
   (b) Show that $T$ is one-to-one on $S^*$.

9. Let $D^*$ be the parallelogram with vertices at $(-1, 3)$, $(0, 0)$, $(2, -1)$, and $(1, 2)$. Let $D$ be the rectangle $D = [0, 1] \times [0, 1]$. Find a $T$ such that $D$ is the image set of $D^*$ under $T$; that is, $D = T(D^*)$.

10. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the spherical coordinate mapping defined by $(\rho, \phi, \theta) \mapsto (x, y, z)$, where

$$x = \rho\sin(\phi)\cos(\theta), \qquad y = \rho\sin(\phi)\sin(\theta), \qquad z = \rho\cos(\phi).$$

Let $D^*$ be the set of points $(\rho, \phi, \theta)$ such that $\rho \in [0, 1]$, $\phi \in [0, \pi]$, $\theta \in [0, 2\pi]$.
   (a) Find $D = T(D^*)$.
   (b) Is $T$ one-to-one? If not, can we eliminate a subset $S \subseteq D^*$ so that $T$ is one-to-one on the remainder $D^* \setminus S = \{(x, y, z) \in D^* \mid (x, y, z) \notin S\}$?