Math 142A **Midterm Exam 2 v. A** **Name:** _____
November 22, 2017 **(Total Points: *25*)** **PID:** _____

Instructions

1. **Write your Name and PID in the spaces provided above.**
2. **Make sure your Name is on every page.**
3. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
4. **Put away ANY devices that can be used for communication or can access the Internet.**
5. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
6. **Read each question carefully and answer each question completely.**
7. **Write your solutions clearly in the spaces provided.**
8. **Show all of your work. No credit will be given for unsupported answers, even if correct.**

(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions given before the exam or written on the chalkboard during the exam.

(6 points) 1. Let $f : (0,1) \to \mathbb{R}$ be given by $f(x) = \cos\left(\frac{1}{x}\right)$. f is continuous since it is the composition of continuous functions and its image $f((0,1))$ is bounded since $|f(x)| \leq 1$ for all x in $(0,1)$.

Show that f is not uniformly continuous.

(6 points) 2. Consider the strictly increasing function

$$f : [0,1) \cup [2,4] \to \mathbb{R}$$
$$f(x) = \begin{cases} x & \text{if } 0 \leq x < 1, \\ x - 1 & \text{if } 2 \leq x \leq 4. \end{cases}$$

(a) Is f continuous? Justify your answer.

(b) Is f^{-1} continuous? Justify your answer.

(6 points) 3. A function $f : \mathbb{R} \to \mathbb{R}$ has the property that there is a number $C > 0$ such that $|f(x)| \leq Cx^2$ for all x.

Prove that $\lim_{x \to 0} \dfrac{f(x)}{x} = 0$.

(6 points) 4. (a) Show that $f(x) = |x|$ is not differentiable at $x = 0$.

(b) Show that if $f : \mathbb{R} \to \mathbb{R}$ is differentiable at x_0, then f is continuous at x_0.